# Revision Booster WORKSHOP for NEET & JEE Main

# Simple Harmonic Motion

Notes of Revision Booster Workshop for JEE Main & NEET

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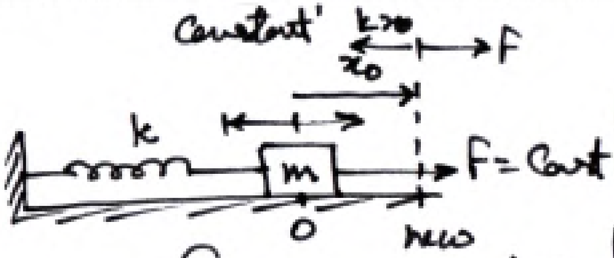
Questions Based on

# # Effect of External Force in SHM




$\left[ F = kx_0 \right]$

$\omega = \sqrt{\frac{k}{m}}$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

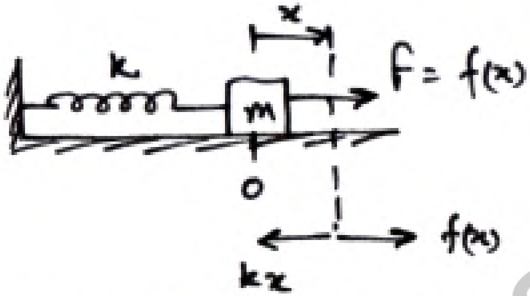
*

(1) changes mean pos of SHM

(2) does not change $\omega$ of SHM

QUESTIONS BASED ON

# # EFFECT OF VARIABLE EXTERNAL FORCE IN SHM




Restoring force $kx - f(x) = ma$

$$a = -\frac{kx - f(x)}{m} = -Cx$$

Comp with $a = -\omega^2 x$

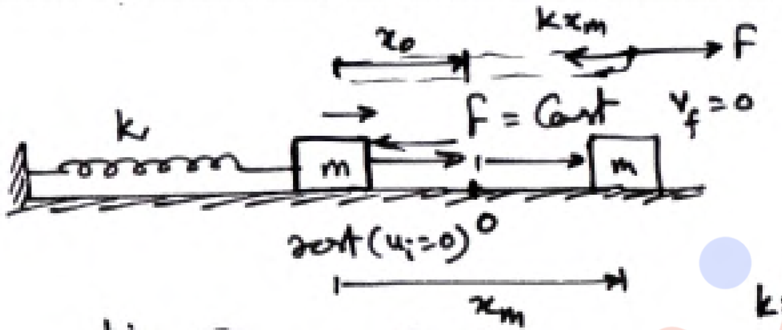
$$\omega = \sqrt{C}$$

By Taylor's th.

$$\omega = \sqrt{\frac{f'(0)}{m}}$$

Questions Based on

# # SHM Amplitude Due to External Force




$$kx_0 = F$$

$$x_0 = F/k$$

$$\overset{K_i}{0} + Fx_m - \frac{1}{2}kx_m^2 = \overset{K_f}{0}$$

$$x_m = \frac{2F}{k}$$

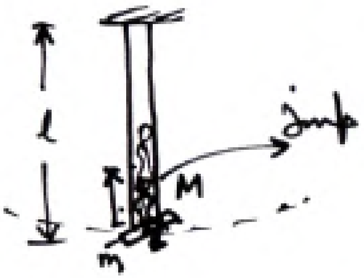
Amp of osc $A = x_m - x_0 = F/k$

if $F = f(x)$

$$W = \int_0^{x_m} f(x)\,dx = \_\_\_\_$$

Questions Based on

# # A Boy Swinging on a Swing




$T = 2\pi\sqrt{\frac{l}{g}}$

$l\downarrow \Rightarrow T\downarrow$ osc. will become faster

if boy jumps $\Rightarrow$ $l_{eff}$ $\rightarrow$ remains same

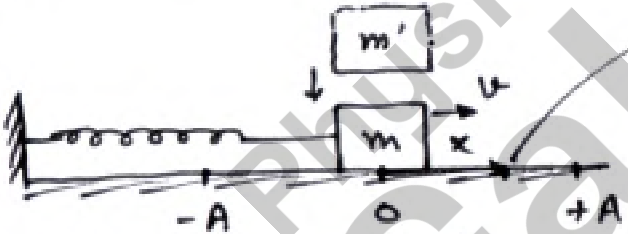
$\Rightarrow T \longrightarrow$ same.

QUESTIONS BASED ON

# # ADDING OR REMOVING MASS FROM A SPRING-BLOCK SYSTEM


$$\frac{1}{2}kA^2 = \frac{1}{2}(m+m')v'^2_{max}$$

$$\omega = \sqrt{\frac{k}{m}} \longrightarrow \omega' = \sqrt{\frac{k}{m+m'}}$$




$$v_x = \omega\sqrt{A^2 - x^2} = \omega'\sqrt{A'^2 - x^2}$$

$$\omega_{new} = \sqrt{\frac{k}{m+m'}}$$

$A' = \_\_\_$ ✓

$$v_{max} = A\omega$$

$$mv_{max} = (m+m')v'_{max}$$ ✓

$$v'_{max} = \frac{mv}{(m+m')}$$

$$\frac{1}{2}(m+m')v'^2_{max} = \frac{1}{2}kA'^2$$

$A' = \_\_\_$ new amp of SHM.

QUESTIONS BASED ON

# # AMPLITUDE OF A GIVEN SHM EQUATION

$y = A \sin(\omega t + \alpha)$
↳ amp


if at $t=0$ particle is moving away from mean pos $\Rightarrow \alpha < 90$ or $180° < \alpha < 270°$

" " " " towards " " $\Rightarrow 180° < \alpha > 90°$ or $270° < \alpha < 360°$

or $y = A_1 \sin \omega t + A_2 \cos \omega t$

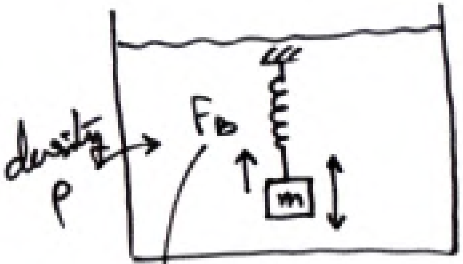
$A_1 \downarrow R\cos\theta$, $A_2 \downarrow R\sin\theta$

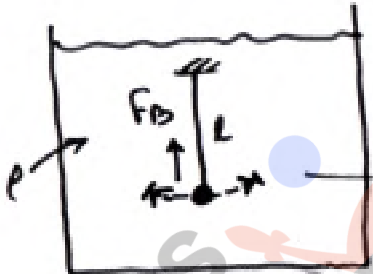
$\left.\begin{array}{l} R\sin\theta = A_2 \\ R\cos\theta = A_1 \end{array}\right\} \Rightarrow R = \sqrt{A_1^2 + A_2^2}$, $\theta = \tan^{-1}(A_2/A_1)$

$\Rightarrow y = R \sin(\omega t + \theta)$
↳ amp of Osc.

Questions Based on

# # SHM Submerging in a Liquid




$\longrightarrow g_{eff} = g\left(1 - \frac{\rho}{\rho_s}\right) \longrightarrow T = 2\pi\sqrt{\frac{l}{g_{eff}}}$

Const $F_B = mg\left(\frac{\rho}{\rho_s}\right) \longrightarrow$ Constant

$\Rightarrow \quad \omega_f = \omega_i = \sqrt{\frac{k}{m}}$

$\omega_i = \sqrt{\frac{g}{l}}$

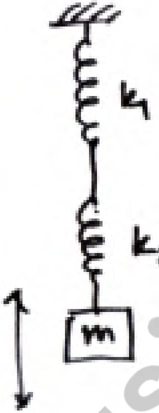
$\omega_f = \sqrt{\frac{g_{eff}}{l}}$

$\Rightarrow$ Osc. will slow down!

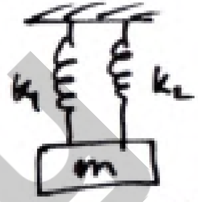
QUESTIONS BASED ON

# SHM WITH CASCADING OF SPRINGS

$T = 2\pi\sqrt{\frac{m}{k}}$




$\longrightarrow k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$ ✓




$k_{eq} = k_1 + k_2$

$T' = 2\pi\sqrt{\frac{m}{k_{eq}}}$

$T_{new} = 2\pi\sqrt{\frac{m}{k_{eq}}}$

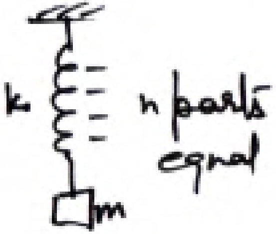
if only $k_1$ $\quad T = T_1 = 2\pi\sqrt{\frac{m}{k_1}} \nearrow \quad k_1 = \frac{4\pi^2 m}{T_1^2}$

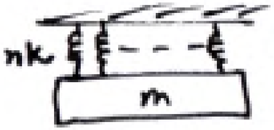
if only $k_2$ $\quad T = T_2 = 2\pi\sqrt{\frac{m}{k_2}} \nearrow \quad k_2 = \frac{4\pi^2 m}{T_2^2}$

$k_1, k_2$ Series $\quad T = $ ______

### Questions Based on

## # SHM With a Part of Spring

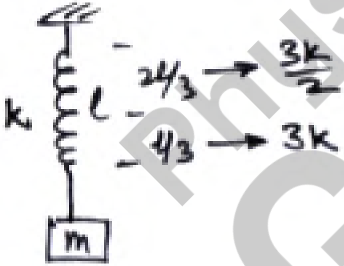


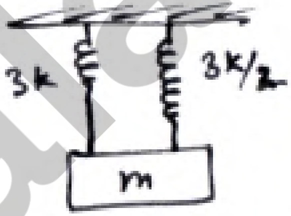




$$\Big]\ k_{eq} = n(nk) = n^2 k$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T_{new} = 2\pi\sqrt{\frac{m}{n^2 k}} = \frac{T}{n}$$



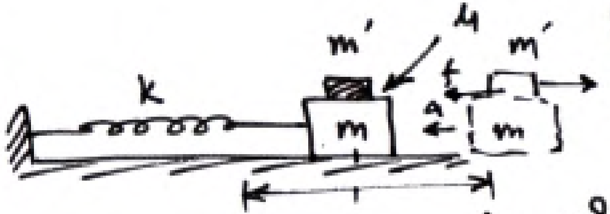




$$k_{eq} = 3k + \frac{3k}{2} = \frac{9k}{2}$$

$$T = 2\pi\sqrt{\frac{2m}{9k}}$$ ✓

QUESTIONS BASED ON

# # SLIDING OF A BODY IN SHM




$A_{max} = ?$ for $m'$ not to slide

$a_{max} = \omega^2 A_{max}$

for $m'$ not to slide

$\Rightarrow \quad m'\omega^2 A_{max} < \mu m' g$

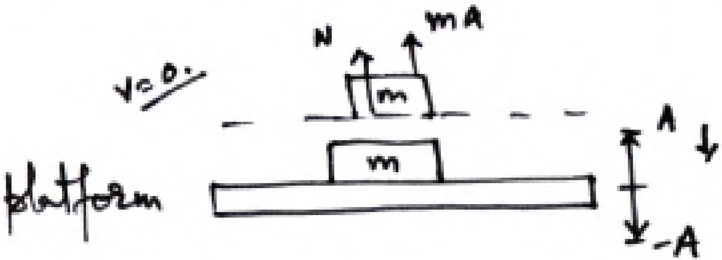
$A_{max} < \dfrac{\mu g}{\omega^2}$ ✓

$\omega = \sqrt{\dfrac{k}{m+m'}}$

Questions Based on

# # Breaking off From Platform in SHM

find $A_{max}$ for m not to break off




$\downarrow a_{max} = \omega^2 A$

$\omega$

if $ma \geq mg \Rightarrow N = 0$

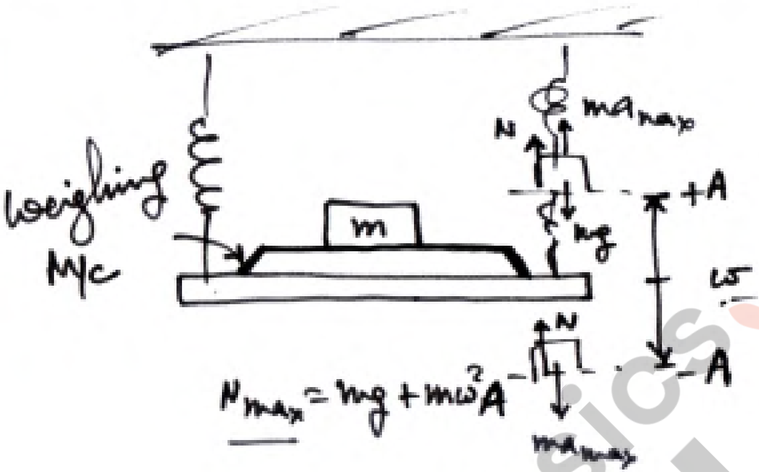
$a \geq g \Rightarrow$ breaking off

$\omega^2 A \geq g$

max $A \geq g/\omega^2$

## Questions Based on

# # Maximum and Minimum Weight in Weighing Machine




$$\omega = \sqrt{\frac{k}{m + M_{p+wm}}}$$

$$\underline{N_{min}} = mg - m\omega^2 A = 0$$

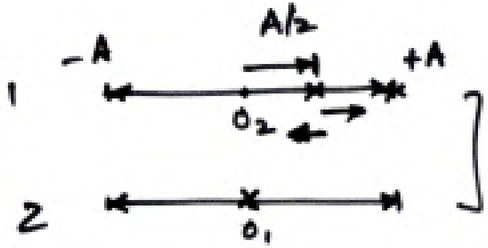
$$W_{meas\ min} = \frac{N_{min}}{g}\ \text{kgf}$$

$$\underline{N_{max}} = mg + m\omega^2 A$$

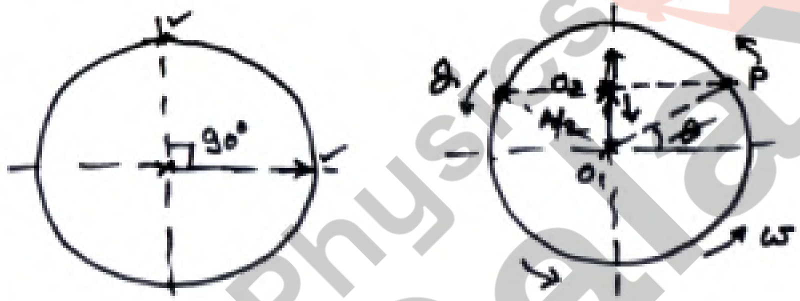
$$W_{meas\ max} = \frac{N}{g}\ \text{kgf.}$$

Questions Based on

# # Phase Difference Between Two SHMs




$\omega \rightarrow$ Same $\Rightarrow$ Phase diff $\rightarrow$ Const




$\phi_P = \sin^{-1}(1/2) = \pi/6$ ✓

$\phi_Q = \pi - \pi/6 = \frac{5\pi}{6}$ ✓

QUESTIONS BASED ON

# # KINETIC & POTENTIAL ENERGY VARIATION IN SHM

if $x = A\sin(\omega t+\phi)$

$$K_x = \frac{1}{2}m\omega^2(A^2-x^2) = \frac{1}{2}m\omega^2A^2\underline{\cos^2(\omega t+\phi)} \longrightarrow K_{avg} = \frac{1}{4}m\omega^2A^2 = P_{avg}$$

$$P_x = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2A^2\underline{\sin^2(\omega t+\phi)}$$

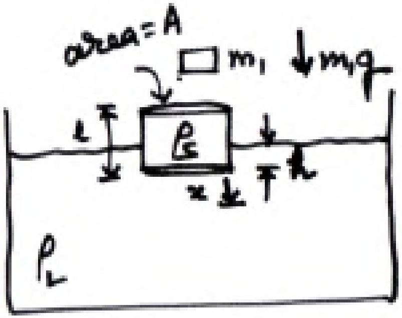
$$E = \underline{\frac{1}{2}m\omega^2A^2} = c$$

$$\cos 2(\omega t+\phi) = 2\cos^2(\omega t+\phi) - 1$$

$$K_t = \frac{1}{2}m\omega^2A^2\left(\frac{1+\cos 2(\omega t+\phi)}{2}\right) \longrightarrow \text{variation freq of } K \,\&\, P \rightarrow 2\times \text{freq of SHM.}$$

Questions Based on

# # Wooden Block Floating in a Liquid Executing SHM




at eq$^m$

$$lA\rho_s g = hA\rho_L g \quad \text{—(1)}$$

Restoring force

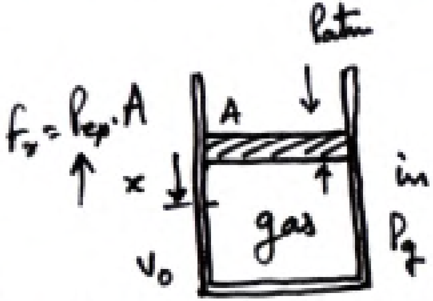
$$-\rho_L x A g = ma$$

$$\Rightarrow a = -\underbrace{\left(\frac{\rho_L A g}{m}\right)}_{\omega^2} x$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m(+m_1)}{A\rho_L g}}$$

**QUESTIONS BASED ON**

## # SHM OF A PISTON ABOVE A TRAPPED GAS




in eq$^m$ $\Rightarrow P_g = P_{atm}$

Say isothermal compression is taken

✓ $P_{atm} \cdot V_0 = P_f (V_0 - Ax)$

$$\Rightarrow \quad P_f = \frac{P_{atm} \cancel{V_0}}{\cancel{V_0}\left(1 - \frac{Ax}{V_0}\right)} = P_{atm}\left(1 + \frac{Ax}{V_0}\right)$$

$P_f - P_{atm}$

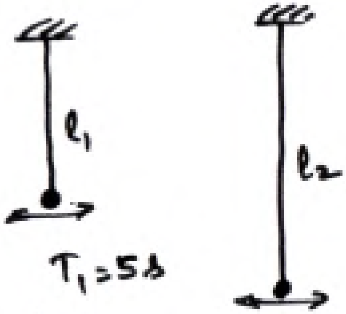
Restoring force on piston upward $\quad F_R = \underbrace{-\frac{P_{atm}A}{V_0}}\cdot x$

$$a = -\left(\frac{P_{atm}A}{mV_0}\right)x \longrightarrow \omega^2$$

Questions Based on

# Same Phase Period of Two Pendulums

$T = 2\pi\sqrt{l/g}$




$T_1 = 5s$

$t = 0$

$T_2 = 6s$

$t = 0$

$(n+1)$ osc

$n$ osc

$[n(6) = (n+1)5]$

$n = 5$ osc

$\Delta t = 5 \times 6 = 30s.$

after every 30 s both pendulums will be in same phase.

QUESTIONS BASED ON

# AMPLITUDE DECAY IN DAMPED OSCILLATIONS

$$\boxed{A = A_0 e^{-bt/2m}}$$

at $t = 10s \longrightarrow A = 0.8A_0$

at $t = 20s \longrightarrow A = ?$

$$0.8 = e^{-10b/2m}$$

$$A = A_0 e^{-20b/2m}$$

$$\Rightarrow A = A_0(0.64) = 0.64A_0 \text{ Ans}$$

QUESTIONS BASED ON

# # DIFFERENTIAL EQUATION OF FORCED OSCILLATIONS

damping force $F_v = -bv$

diff eq$^n$

$$m\frac{d^2x}{dt^2} = -bv - m\omega^2 x$$

$$\Rightarrow \frac{d^2x}{dt^2} + \frac{b}{m}v + \omega^2 x = 0$$

Sol$^n$ $\longrightarrow$ $x = A_0 e^{-bt/2m} \sin(\omega t + \phi)$

# Average lifetime of damping

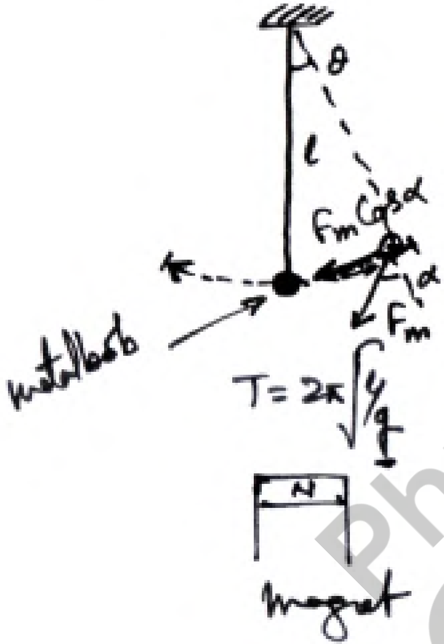
at $t = \tau \longrightarrow A = A_0/e$

$$\Rightarrow \frac{A_0}{e} = A_0 e^{-b\tau/2m}$$

$$\frac{b\tau}{2m} = 1 \Rightarrow \boxed{\tau = \frac{2m}{b}}$$

**QUESTIONS BASED ON**

# # DAMPING OF SIMPLE PENDULUM BY EXTERNAL FORCES




In such cases

effective value of $g \uparrow \Rightarrow T \downarrow$

$\Rightarrow$ pendulum osc will be faster

Here $F_m$ also act as a damping force.

QUESTIONS BASED ON

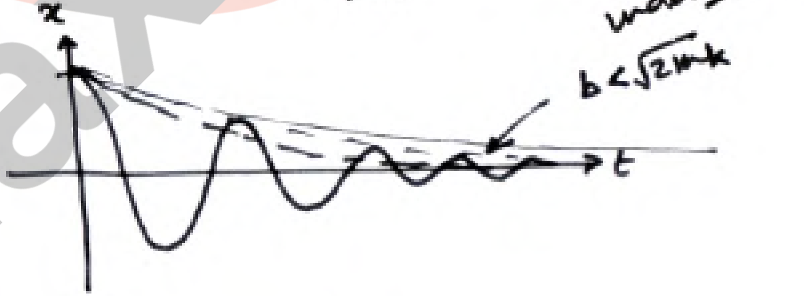
# # OVERDAMPED OR CRITICALLY DAMPED OSCILLATIONS

Differential eqⁿ of damped SHM -

$$m\frac{d^2x}{dt^2} + bv + kx = 0$$

here natural freq of SHM

$$\omega_0 = \sqrt{\frac{k}{m}}$$




Solⁿ to this eqⁿ is

$$x = \underbrace{A_0 e^{-bt/2m}} \sin(\omega t + \phi)$$

here $\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

if $b = \sqrt{2mk} \rightarrow \omega = 0$ [~~Overda~~ Critically damped]

$b > \sqrt{2mk} \rightarrow \omega \rightarrow$ imag [Overdamped Osc

almost no oscillations

Questions Based on

# # Energy in Damped Oscillations

$$A = A_0 e^{-bt/2m}$$

$$\Rightarrow \underline{E_t = E_0 e^{-bt/m}}$$

at $t=0$ : $E = E_0$

at $t = 10s$ : $E = \frac{E_0}{100} \Rightarrow \frac{\cancel{E_0}}{100} = \cancel{E_0} e^{-10b/m}$

$$\frac{10b}{m} = \ln(100)$$

$$b = \frac{m}{10}\ln(100)$$

QUESTIONS BASED ON

# # DAMPING OF SHM IN A LIQUID

if Stoke's law is valid:

here viscous force acts as damping force.

$\rightarrow F_r = -6\pi\eta r v$

$F_r = -bv$

$b = 6\pi\eta r.$

$b \propto \eta$

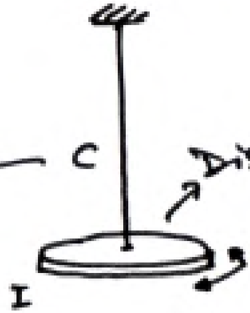
or $\frac{b_1}{b_2} = \frac{\eta_1}{\eta_2}$

QUESTIONS BASED ON

# # ANGULAR SHM OF TORSIONAL PENDULUM

$I\alpha = \tau_R = C\theta$

$\alpha = \left(\frac{C\theta}{I}\right)$




Disc ← rod

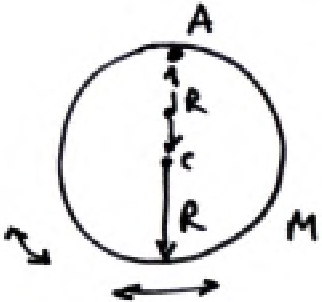
$\omega = \sqrt{\frac{C}{I}} \rightarrow \frac{1}{2}MR^2 \rightarrow \frac{ML^2}{12}$

$\theta_t = \theta_0 \sin(\omega t + \alpha)$


$\sin\alpha = \frac{\theta}{\theta_0}$

QUESTIONS BASED ON

# # ANGULAR SHM OF A HANGING RING




$$T_1 = 2\pi\sqrt{\frac{I}{Mgl}} = 2\pi\sqrt{\frac{2MR^2}{MgR}} = 2\pi\sqrt{\frac{2R}{g}}$$


$$T_2 = 2\pi\sqrt{\frac{3/2\,MR^2}{MgR}} = 2\pi\sqrt{\frac{3R}{2g}}$$

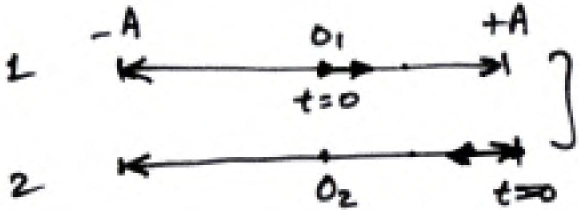
$$\left(l_{eq} = l + \frac{k^2}{l}\right)$$

in Compound pendulum T is min when $l = k$

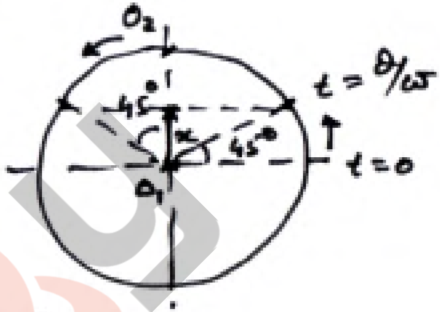
Ring $k = R$ ; Disc $k = R/\sqrt{2}$ - - - -

QUESTIONS BASED ON

# TIME OF CROSSING PARTICLES IN TWO SHMs



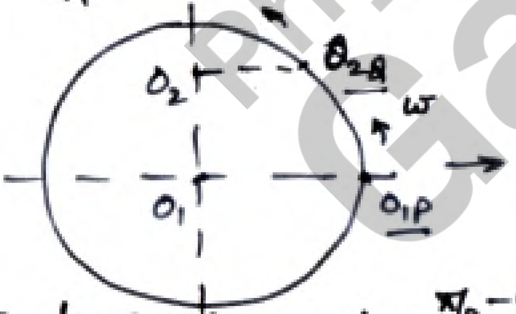

$\Big\}\ \omega \rightarrow$ same



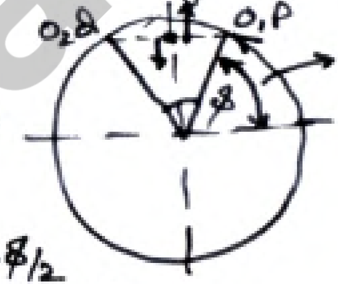

$x = A/\sqrt{2}$ ✓

$$t = \frac{\pi/4}{\omega} = \frac{\pi}{4\omega} \text{ Ans.}$$

if phase diff = $\phi$ in 1 & 2 if 1 is starting from mean pos.




crossing situation




$\theta = \pi/2 - \phi/2$

time to cross $\longrightarrow t = \dfrac{\pi/2 - \phi/2}{\omega}$ ✓