CHAPTER 1

Basic Mathematics Applied in Physics

QUADRATIC EQUATION

Roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sum of roots $x_1 + x_2 = -\dfrac{b}{a}$; Product of roots $x_1x_2 = \dfrac{c}{a}$

BINOMIAL APPROXIMATION

If $x << 1$, then $(1 + x)^n \approx 1 + nx$ & $(1 - x)^n \approx 1 - nx$

LOGARITHM

$\log mn = \log m + \log n$

$\log m/n = \log m - \log n$

$\log m^n = n \log m$

$\log_e m = 2.303 \log_{10} m$

$\log 2 = 0.3010$

COMPONENDO AND DIVIDENDO LAW

If $\dfrac{p}{q} = \dfrac{a}{b}$ then $\dfrac{p+q}{p-q} = \dfrac{a+b}{a-b}$

ARITHMETIC PROGRESSION-AP FORMULA

$a, a + d, a + 2d, a + 3d, \ldots, a + (n - 1)d$, here d = common difference

Sum of n terms $S_n = \dfrac{n}{2}[2a + (n - 1)d]$

NOTES

(i) $1 + 2 + 3 + 4 + 5 \ldots + n = \dfrac{n(n+1)}{2}$

(ii) $1^2 + 2^2 + 3^2 + \ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

GEOMETRICAL PROGRESSION-GP FORMULA

$a, ar, ar^2, \ldots$ here, r = common ratio

Sum of n terms $S_n = \dfrac{a(1-r^n)}{1-r}$

Sum of ∞ terms $S_\infty = \dfrac{a}{1-r}$

TRIGONOMETRY

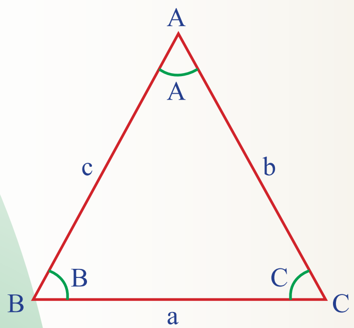
- 2π radian $= 360° \quad \Rightarrow \quad 1$ rad $= 57.3°$
- $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$
- $\sec\theta = \dfrac{1}{\cos\theta}$
- $\cot\theta = \dfrac{1}{\tan\theta}$
- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \operatorname{cosec}^2\theta$
- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
- $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- $\sin 2A = 2 \sin A \cos A$
- $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$
- $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

sine law

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

cosine law

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}, \cos B = \frac{c^2 + a^2 - b^2}{2ca}, \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



Approximation for small θ

- $\sin\theta \approx \theta$
- $\cos\theta \approx 1$
- $\tan\theta \approx \theta \approx \sin\theta$

Differentiation

- $y = x^n \rightarrow \frac{dy}{dx} = nx^{n-1}$
- $y = \ln x \rightarrow \frac{dy}{dx} = \frac{1}{x}$
- $y = \sin x \rightarrow \frac{dy}{dx} = \cos x$
- $y = \cos x \rightarrow \frac{dy}{dx} = -\sin x$
- $y = e^{\alpha x+\beta} \rightarrow \frac{dy}{dx} = \alpha e^{\alpha x+\beta}$
- $y = uv \rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ (Product rule)
- $y = f(g(x)) \rightarrow \frac{dy}{dx} = \frac{df(g(x))}{dg(x)} \times \frac{d(g(x))}{dx}$ (Chain rule)
- $y = k = \text{constant} \Rightarrow \frac{dy}{dx} = 0$
- $y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ (Division Rule)

Integration

- $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
- $\int \frac{1}{x} dx = \ell nx + C$
- $\int \sin x dx = -\cos x + C$
- $\int \cos x dx = \sin x + C$
- $\int e^{\alpha x+\beta} dx = \frac{1}{\alpha} e^{\alpha x+\beta} + C$
- $\int (\alpha x+\beta)^n dx = \frac{(\alpha x+\beta)^{n+1}}{\alpha(n+1)} + C$

Maxima and Minima of a Function y = f(x)

- For maximum value $\frac{dy}{dx} = 0$ & $\frac{d^2y}{dx^2} = -ve$
- For minimum value $\frac{dy}{dx} = 0$ & $\frac{d^2y}{dx^2} = +ve$

Average of a Varying Quantity

If y = f(x) then $< y > = \bar{y} = \frac{\int_{x_1}^{x_2} ydx}{\int_{x_1}^{x_2} dx} = \frac{\int_{x_1}^{x_2} ydx}{x_2 - x_1}$

Formulae for Calculation of Area

- Area of a square = $(\text{side})^2$
- Area of rectangle = length × breadth
- Area of a triangle = 1/2 × base × height
- Area of a trapezoid = 1/2 × (distance between parallel sides) × (sum of parallel sides)
- Area enclosed by a circle = πr^2 (r = radius)
- Surface area of a sphere = $4\pi r^2$ (r = radius)
- Area of a parallelogram = base × height

- Area of curved surface of cylinder = $2\pi rl$ (r = radius and l = length)
- Area of whole surface of cylinder = $2\pi r(r + l)$ (l = length)
- Area of ellipse = π ab (a & b are semi major and semi minor axis repsectively).
- Surface area of a cube = $6(\text{side})^2$

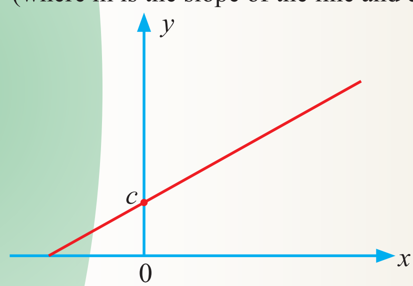
Volume of Geometrical Figures

- Volume of a sphere = $\frac{4}{3}\pi r^3$ (r = radius)
- Volume of a cylinder = $\pi r^2 l$ (r = radius and l = length)
- Volume of a cone = $\frac{1}{3}\pi r^2 h$ (r = radius and h = height)

Some Basic Plots

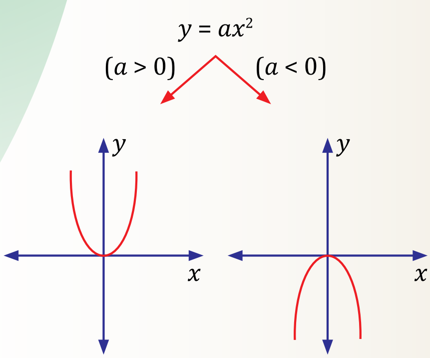
- **Straight Line**

  $y = mx + c$ (where m is the slope of the line and c is the y intercept)



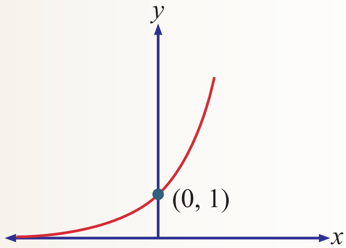
- **Parabola**

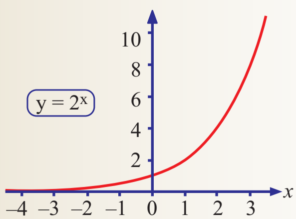
  $\mathbf{y = ax^2}$

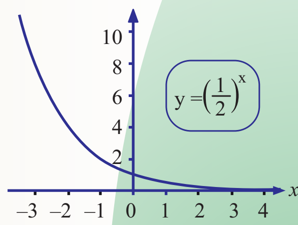


- **Exponential function**

$y = e^x$







KEY TIPS

- To convert an angle from degree to radian, we should multiply it by $\pi/180°$ and to convert an angle from radian to degree, we should multiply it by $180°/\pi$.
- By help of differentiation, if y is given, we can find dy/dx and by help of integration, if dy/dx is given, we can find y.
- The maximum and minimum values of function $A\cos\theta + B\sin\theta$ are $\sqrt{A^2+B^2}$ and $-\sqrt{A^2+B^2}$ respectively.
- $(a + b)^2 = a^2 + b^2 + 2ab$
- $(a - b)^2 = a^2 + b^2 - 2ab$
- $(a + b)(a - b) = a^2 - b^2$
- $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
- $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

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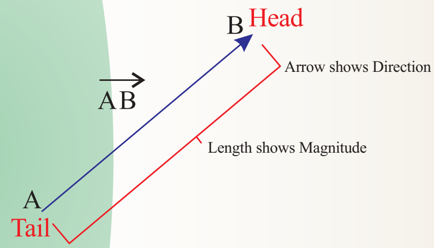
CHAPTER 2

Vectors

VECTOR QUANTITIES

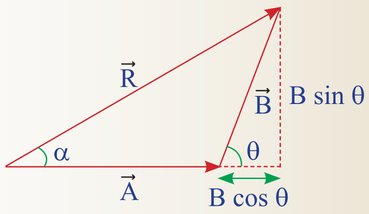
A physical quantity which requires magnitude and a particular direction for its complete expression.



Triangle Law of Vector Addition $\boxed{\vec{R} = \vec{A} + \vec{B}}$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

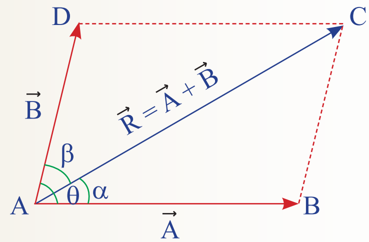


$$\text{If } A = B \text{ then } R = 2A\cos\frac{\theta}{2} \;\&\; \alpha = \frac{\theta}{2}$$

$$R_{max} = A + B \text{ for } \theta = 0°;\; R_{min} = A - B \text{ for } \theta = 180°$$

Parallelogram Law of Vector Addition

If two vectors are represented by two adjacent sides of a parallelogram which are directed away from their common point then their sum (i.e. resultant vector) is given by the diagonal of the paralellogram passing away through that common point.



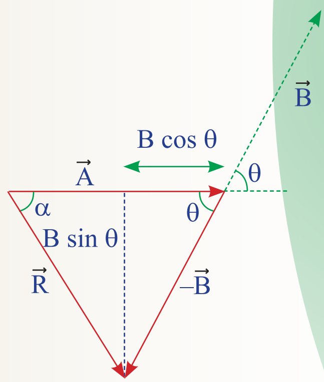
$$\overline{AB}+\overline{AD}=\overline{AC}=\overline{R} \text{ or } \vec{A}+\vec{B}=\vec{R} \Rightarrow R=\sqrt{A^2+B^2+2AB\cos\theta}$$

$$\tan\alpha=\frac{B\sin\theta}{A+B\cos\theta} \text{ and } \tan\beta=\frac{A\sin\theta}{B+A\cos\theta}$$

Vector Subtraction

$$\boxed{\vec{R}=\vec{A}-\vec{B}} \quad \Rightarrow \quad \vec{R}=\vec{A}+(-\vec{B})$$

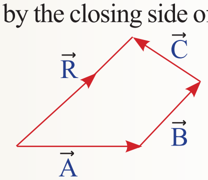
$$R=\sqrt{A^2+B^2-2AB\cos\theta},\ \tan\alpha=\frac{B\sin\theta}{A-B\cos\theta}$$



If $A=B$ then $R=2A\sin\dfrac{\theta}{2}$

Addition of More than Two Vectors (Polygon Law)

If some vectors are represented by sides of a polygon in same order, then their resultant vector is represented by the closing side of polygon in the opposite order.



Rectangular Components of a 3-D Vector

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

- Angle made with x-axis $\cos\alpha = \dfrac{A_x}{A} = \dfrac{A_x}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = l$
- Angle made with y-axis $\cos\beta = \dfrac{A_y}{A} = \dfrac{A_y}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = m$
- Angle made with z-axis $\cos\gamma = \dfrac{A_z}{A} = \dfrac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = n$

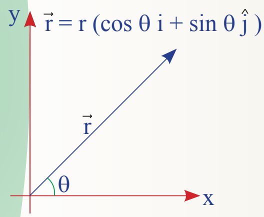
l, m, n are called direction cosines of vector $\vec{A}$.

$$l^2 + m^2 + n^2 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{A_x^2 + A_y^2 + A_z^2}{(\sqrt{A_x^2 + A_y^2 + A_z^2})} = 1$$

or $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

General Vector in x-y Plane

$\vec{r} = x\hat{i} + y\hat{j} = r(\cos\theta\hat{i} + \sin\theta\hat{j})$



Scalar Product (Dot Product)

$\vec{A}\cdot\vec{B} = AB\cos\theta \Rightarrow \boxed{\text{Angle between two vectors} = \cos^{-1}\left(\dfrac{\vec{A}\cdot\vec{B}}{AB}\right)}$

e.g. work done $= \vec{F}\,.\,\vec{S}$ (where $\vec{F}$ is the Force vector & $\vec{S}$ is the displacement vector).

If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ & $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ then

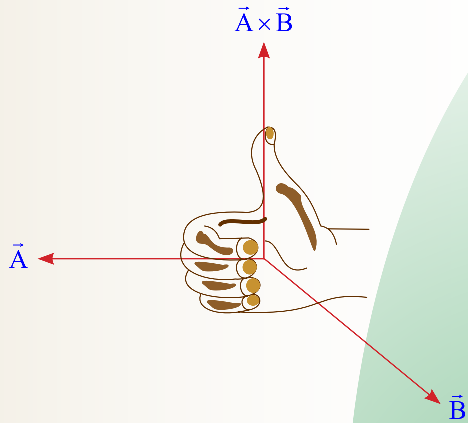
$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$ and angle between $\vec{A}$ & $\vec{B}$ is given by

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\,\sqrt{B_x^2 + B_y^2 + B_z^2}}$$

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1,$$
$$\hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = 0$$

Cross Product (Vector Product)

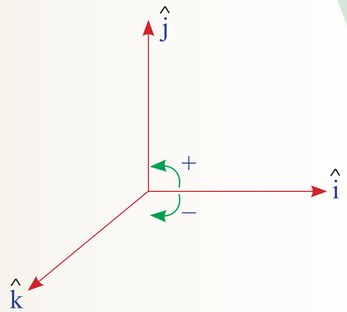
$\vec{A}\times\vec{B} = AB\sin\theta\ \hat{n}$ where $\hat{n}$ is a vector perpendicular to $\vec{A}$ & $\vec{B}$ or their plane and its direction given by right hand thumb rule.



- $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$
- $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

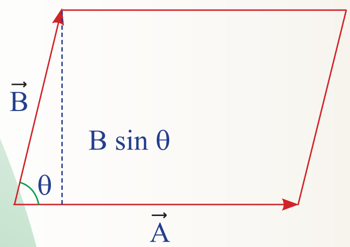
$$= \hat{i}(A_yB_z - A_zB_y) - \hat{j}(A_xB_z - B_xA_z) + \hat{k}(A_xB_y - B_xA_y)$$

- $\hat{i}\times\hat{i} = \vec{0},\ \hat{j}\times\hat{j} = \vec{0},\ \hat{k}\times\hat{k} = \vec{0}$
- $\hat{i}\times\hat{j} = \hat{k};\ \hat{j}\times\hat{k} = \hat{i},$
  $\hat{k}\times\hat{i} = \hat{j};\ \hat{j}\times\hat{i} = -\hat{k}$
  $\hat{k}\times\hat{j} = -\hat{i},\ \hat{i}\times\hat{k} = -\hat{j}$



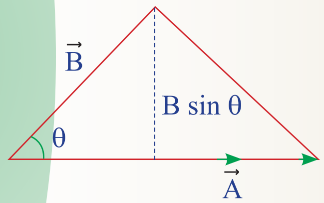
Area of Parallelogram

$\overrightarrow{\text{Area}} = \left(|\vec{A}||\vec{B}|\sin\theta\right)\hat{n} = \vec{A}\times\vec{B}$ (where $\hat{n}$ is the unit vector normal to the plane containing $\vec{A}$ and $\vec{B}$)



Area of Triangle

$$\text{Area} = \frac{|\vec{A}\times\vec{B}|}{2} = \frac{1}{2}AB\sin\theta$$



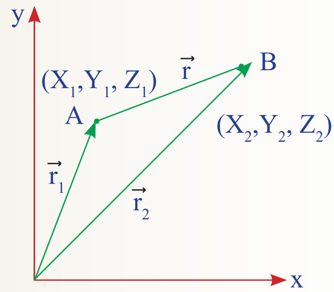
Differentiation of Vectors

- $\frac{d}{dt}(\vec{A}\cdot\vec{B}) = \frac{dA}{dt}\cdot\vec{B} + \vec{A}\cdot\frac{d\vec{B}}{dt}$

- $\frac{d}{dt}(\vec{A}\times\vec{B}) = \frac{d\vec{A}}{dt}\times\vec{B} + \vec{A}\times\frac{d\vec{B}}{dt}$

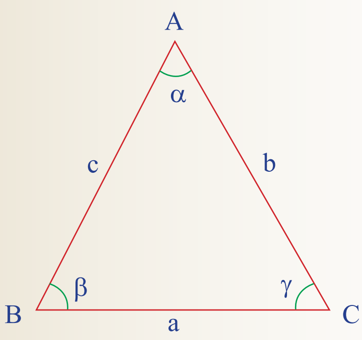
Displacement Vector

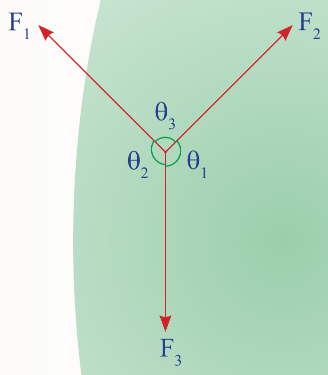
$$\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$$
$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$



Magnitude $r = |\vec{r}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Lami's Theorem





$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

KEY TIPS

- A unit vector has no unit.
- Electric current is not a vector as it does not obey the law of vector addition.
- A scalar or a vector can never be divided by a vector.
- To a vector only a vector of same type can be added and the resultant is a vector of the same type.

CHAPTER 3

Units, Dimensions and Measurements

SYSTEMS OF UNITS

S.No.	MKS	CGS	FPS
1.	Length (m)	Length (cm)	Length (ft)
2.	Mass (kg)	Mass (g)	Mass (pound)
3.	Time (s)	Time (s)	Time (s)

Fundamental Quantities in S.I. System and Their Units

S.No.	Physical Quantity	Name of Unit	Symbol
1.	Mass	kilogram	kg
2.	Length	meter	m
3.	Time	second	s
4.	Temperature	kelvin	K
5.	Luminous intensity	candela	Cd
6.	Electric current	ampere	A
7.	Amount of substance	mole	mol

Dimensional Formula

Relations which express physical quantities in terms of appropriate powers of fundamental units.

Application of Dimensional Analysis

- To check the dimensional consistency of a given physical relationship.
- To derive relationship between various physical quantities.
- To convert units of a physical quantity from one system to another

$$n_1u_1 = n_2u_2 \Rightarrow n_2 = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c \text{ where } u = M^aL^bT^c$$

Dimensional Formulae of Various Physical Quantities

S.No.	Physical Quantity	Dimensional Formula
	Mechanics	
1.	Area	$L \times L = L^2 = [M^0L^2T^0]$
2.	Volume	$L \times L \times L = [M^0L^3T^0]$
3.	Density	$\frac{M}{L^3} = [ML^{-3}T^0]$
4.	Speed or Velocity	$\frac{L}{T} = [M^0LT^{-1}]$
5.	Acceleration	$\frac{LT^{-1}}{T} = LT^{-2} = [M^0LT^{-2}]$
6.	Momentum	$M \times LT^{-1} = [MLT^{-1}]$
7.	Force	$M \times LT^{-2} = [MLT^{-2}]$
8.	Work	$MLT^{-2} \times L = [ML^2T^{-2}]$
9.	Energy	$[ML^2T^{-2}]$
10.	Power	$\frac{ML^2T^{-2}}{T} = [ML^2T^{-3}]$
11.	Pressure	$\frac{ML^1T^{-2}}{L^2} = [ML^{-1}T^{-2}]$
12.	Moment of force or torque	$MLT^{-2} \times L = [ML^2T^{-2}]$
13.	Gravitational constant ‘G’	$\frac{[MLT^{-2}][L^2]}{M \times M} = [M^{-1}L^3T^{-2}]$
14.	Impulse of a force	$MLT^{-2} \times T = [MLT^{-1}]$
15.	Stress	$\frac{MLT^{-2}}{L^2} = [ML^{-1}T^{-2}]$
16.	Strain	$[M^0L^0T^0]$

S.No.	Physical Quantity	Dimensional Formula
17.	Coefficient of elasticity	$\frac{MLT^{-2}}{L^2} = [ML^{-1}T^{-2}]$
18.	Surface tension	$\frac{MLT^{-2}}{L} = MT^{-2} = [ML^0T^{-2}]$
19.	Free Surface energy	$\frac{ML^2T^{-2}}{L^2} = [ML^0T^{-2}]$
20.	Coefficient of viscosity	$\frac{MLT^{-2} \times L}{L^2 \times LT^{-1}} = [ML^{-1}T^{-1}]$
21.	Angle	$\frac{L}{L} = 1 = [M^0L^0T^0]$
22.	Angular velocity	$\frac{1}{T} = T^{-1} = [M^0L^0T^{-1}]$
23.	Angular acceleration	$\frac{T^{-1}}{T} = T^{-2} = [M^0L^0T^{-2}]$
24.	Moment of inertia	$ML^2 = [ML^2T^0]$
25.	Radius of gyration	$L = [M^0LT^0]$
26.	Angular momentum	$M \times LT^{-1} \times L = [ML^2T^{-1}]$
27.	T-ratios (sin θ, cos θ, tan θ)	$\frac{L}{L} = 1 = [M^0L^0T^0]$ (dimensionless)
28.	Time period	$T = [M^0L^0T^{-1}]$
29.	Frequency	$\frac{1}{T} = T^{-1} = [M^0L^0T^{-1}]$
30.	Planck's constant 'h'	$\frac{ML^2T^{-2}}{T^{-1}} = [ML^2T^{-1}]$
31.	Relative density	$\frac{ML^{-3}}{ML^{-3}} = 1 = [M^0L^0T^0]$ (Dimensionless)

S.No.	Physical Quantity	Dimensional Formula
32.	Velocity gradient	$\frac{LT^{-1}}{L} = T^{-1} = [M^0L^0T^{-1}]$
33.	Pressure gradient	$\frac{ML^{-1}T^{-2}}{L} = [ML^{-2}T^{-2}]$
34.	Force constant	$\frac{MLT^{-2}}{L} = MT^{-2} = [ML^0T^{-2}]$
Thermodynamics		
35.	Heat or enthalpy	$[ML^2T^{-2}]$
36.	Specific heat	$\frac{[ML^2T^{-2}]}{[M][K]} = [M^0L^2T^{-2}K^{-1}]$
37.	Latent heat	$\frac{[ML^2T^{-2}]}{[M]} = [M^0L^2T^{-2}]$
38.	Thermal conductivity	$\frac{ML^2T^{-2}.L}{L^2.K.T} = [MLT^{-3}K^{-1}]$
39.	Entropy	$\frac{ML^2T^{-2}}{K} = [ML^2T^{-2}K^{-1}]$
40.	Universal Gas Constant	$\frac{ML^{-1}T^{-2}L^3}{mol.K} = [ML^2T^{-2}K^{-1}mol^{-1}]$
41.	Thermal conductivity	$\frac{[ML^2T^2.L]}{[L^2.K.T]} = [MLT^{-3}K^{-1}]$
42.	Universal Gas Constant	$\frac{[ML^{-1}T^{-2}[L^3]}{[mol.K]} = [ML^2T^{-3}K^{-1}mol^{-1}]$
43.	Boltzmann's Constant	$\frac{[ML^2T^{-2}]}{[K]} = [ML^2T^{-2}K^{-1}]$
44.	Stefan's constant	$\frac{[ML^2T^{-2}]}{[L^2.T.K^4]} = [ML^0T^{-3}K^{-4}]$

S.No.	Physical Quantity	Dimensional Formula
45.	Solar constant	$\frac{[ML^2T^{-2}]}{[L^2.T]}=[ML^0T^{-3}]$
46.	Mechanical equivalent of heat	$\frac{[ML^2T^{-2}]}{[ML^2T^{-2}]}=[M^0L^0T^0]$ (Dimensionless)
	Electrostats	
47.	Electric Charge	$[T.A] = [M^0L^0TA]$
48.	Electrical potential	$\frac{[ML^2T^{-2}]}{[TA]}=[ML^2T^{-3}A^{-1}]$
49.	Resistance	$\frac{[ML^2T^{-3}A^{-1}]}{[A]}=[ML^2T^{-3}A^{-2}]$
50.	Capacitance	$\frac{[TA]}{[ML^2T^{-3}A^{-1}]}=[M^{-1}L^{-2}T^4A^2]$
51.	Inductance	$\frac{[ML^2T^{-3}A^{-1}]}{[AT^{-1}]}=[ML^2T^{-2}A^{-2}]$
52.	Permittivity of free space	$\frac{[AT.AT]}{[MLT^{-2}][L^2]}=[M^{-1}L^{-3}T^4A^2]$
53.	Relative permittivity or dielectric constant	A pure ratio = $[M^0L^0T^0]$ (dimensionless)
54.	Intensity of electric field	$\frac{[MLT^{-2}]}{[AT]}=[MLT^{-3}A^{-1}]$
55.	Conductance	$\frac{1}{ML^2T^{-3}A^{-2}}=[M^{-1}L^{-2}T^{-3}A^2]$
56.	Specific resistance or resistivity	$\frac{[ML^2T^{-3}A^{-2}][L^2]}{[L]}=[ML^3T^{-3}A^{-2}]$
57.	Specific conductance of conductivity	$[M^{-1}L^{-3}T^3A^2]$
58.	Electric dipole moment	$[AT][L] = [M^0LTA]$

S.No.	Physical Quantity	Dimensional Formula
Magnetism		
59.	Magnetic field	$\frac{MLT^{-2}}{AT.LT^{-1}.1}=[ML^0T^{-2}A^{-1}]$
60.	Magnetic flux	$[MT^{-2}A^{-1}].[L^2]=[ML^2T^{-2}A^{-1}]$
61.	Permeability of free space	$\frac{[L][MLT^{-2}]}{[A^2.L]}=[MLT^{-2}A^{-2}]$
62.	Magnetic moment	$A.L^2=[M^0L^2T^0A]$
63.	Pole strength	$\frac{AL^2}{L}=[M^0LT^0A]$

- Distance of an object by parallax method, $D=\frac{\text{Basis}}{\text{Parallax angle}}$
- Absolute error = True value – Measured value = $[\Delta a_n]$
- True value = Arithmetic mean of the measured values

$$a_{mean}=\frac{a_1+a_2+\ldots+a_n}{n}$$

- Relative error in the measurement of a quantity = $\frac{\Delta a_{maean}}{a_{mean}}$
- Percentage error = $\frac{\Delta a_{maean}}{a_{mean}}\times 100$
- Maximum permissible error in addition or subtraction of two quantities $(A\pm\Delta A)$ and $(B\pm\Delta B)$: $\Delta A+\Delta B$.
- Maximum permissible relative error in multiplication or division of two quantities $(A\pm\Delta A)$ and $(B\pm AB)$:

$$\frac{\Delta A}{A}+\frac{\Delta B}{B}$$

- When $z=\frac{a^p.b^q}{c^r}$, then maximum relative error in z is

$$\frac{\Delta z}{z}=p\frac{\Delta a}{a}+q\frac{\Delta b}{b}+r\frac{\Delta C}{C}$$

SIGNIFICANT FIGURES

The following rules are observed in counting the number of significant figures in a given measured quantity:

Example: 42.3 has three significant figures.
243.4 has four significant figures.

A zero becomes a significant figure if it appears between two non-zero digits.

Example: 5.03 has three significant figures.
5.604 has four significant figures.

Leading zeros or the zeros placed to the left of the number are never significant.

Example: 0.543 has three significant figures.
0.006 has one significant figure.

Trailing zeros or the zeros placed to the right of the number are significant.

Example: 4.330 has four significant figures.
433.00 has five significant figures.

In exponential notation, the numerical portion gives the number of significant figures.

Example: 1.32×10^{-2} has three significant figures.
1.32×10^{4} has three significant figures.

ROUNDING OFF

While rounding off measurements, we use the following rules by convention:

If the digit to be dropped is less than 5, then the preceding digit is left unchanged.

Example: x = 7.82 is rounded off to 7.8, again x = 3.94 is rounded off to 3.9.

If the digit to be dropped is more than 5, then the preceding digit is raised by 1.

Example: x = 6.87 is rounded off to 6.9, again x = 12.78 is rounded off to 12.8.

If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by 1.

Example: x = 16.351 is rounded off to 16.4, again x = 6.758 is rounded off to 6.8.

If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is left unchanged, if it is even.

Example: x = 3.250 becomes 3.2 on rounding off. again x = 12.650 becomes 12.6 on rounding off.

If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by 1, if it is odd.

Example: x = 3.750 is rounded off to 3.8. again x = 16.150 is rounded off to 16.2.

Significant Figures in Calculation

The following two rules should be followed to obtain the proper number of significant figures in any calculation.

1. The result of an addition or subtraction in the number having different precisions should be rounded off the same number of decimal places as are present in the number having the least number of decimal places. The rule is illustrated by the following examples:

(*a*)

$$\begin{array}{r} 33.3 \\ 3.11 \\ +\,0.313 \\ \hline 36.723 \\ \hline \end{array}$$

(has only one decimal place)

(answer should be rounded off one decimal place)

Answer = 36.7

(*b*)

$$\begin{array}{r} 3.1421 \\ 0.241 \\ +\,0.09 \\ \hline 3.4731 \\ \hline \end{array}$$

(has 2 decimal places)

(answer should be rounded off 2 decimal places)

Answer = 3.47

(*c*)

$$\begin{array}{r} 62.831 \\ -\,24.5492 \\ \hline 38.2818 \\ \hline \end{array}$$

(has 3 decimal places)

(answer should be rounded off 3 decimal places)

Answer = 38.282

2. The answer to a multiplication or division is rounded off to the same number of significant figures as is possessed by the least precise term used in the calculation. The rule is illustrated by the following examples:

(*a*)

$$\begin{array}{r} 142.06 \\ \times\,0.23 \\ \hline 32.6738 \\ \hline \end{array}$$

(two significant figures)

(answer should have two significant figures)

Answer = 33

(*b*)

$$\begin{array}{r} 51.028 \\ \times\,1.31 \\ \hline 66.84668 \\ \hline \end{array}$$

(three significant figures)

Answer = 66.8

(*c*) $\frac{0.90}{4.26} = 0.2112676$

Answer = 0.21

Order of Magnitude

In scientific notation, the numbers are expressed as: Number = M $\times$ 10^3. Where M is a number that lies between 1 and 10 and x is an integer. The order of magnitude of quantity is the power of 10 required to represent the quantity. For determining this power, the value of the quantity has to be rounded off. While rounding off. We ignore the last digit which is less than 5. If the last digit is 5 or more than five, the preceding digit is increased by 1. For example.

Speed of light in vacuum = 3×10^8 ms^{-1} $\approx 10^8$ ms^{-1} (ignoring 3 < 5)

Mass of electron = 9.1×10^{-31} kg $\approx 10^{-30}$ kg (as 9.1 > 5).

Example 1: Each side of a cube is measured to be 7.203 m. Find the volume of the cube up to appropriate significant figures.

Sol. Volume = $a^3 = (7.023)^3 = 373.715$ m^3

Example 2: The mass of abox is 2.3 kg. Two marbles of masses 2.15 g and 12.39 g are added to it. Find the total mass of the box to the correct number of significant figures.

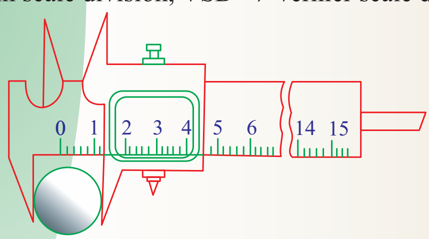
Sol. Total mass = 2.3 + 0.00215 + 0.01239 = 2.31 kg

The total mass in appropriate significant figures will be 2.3 kg.

Vernier scale and screw gauge basics:

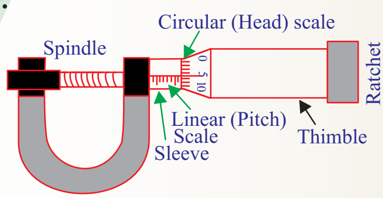
- **Vernier Callipers Least count = 1MSD – 1VSD**

  (MSD $\rightarrow$ main scale division, VSD $\rightarrow$ Vernier scale division)



Ex. A vernier scale has 10 parts, which are equal to 9 parts of main scale having each path equal to 1 mm then least count = 1 mm – $\frac{9}{10}$ mm = 0.1 mm [$\because$ 9 MSD = 10 VSD]

- **Screw Gauge:**



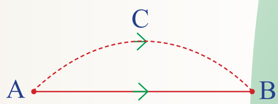
$$\text{Least count} = \frac{\text{pitch}}{\text{total no. of divisions on circular scale}}$$

CHAPTER 4

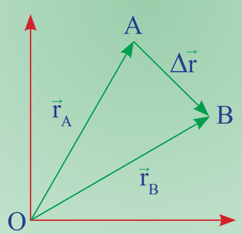
Kinematics

DISTANCE versus DISPLACEMENT

Total length of path (ACB) covered by the particle is called distance. Displacement vector or displacement is the minimum distance (AB) and directed from initial position to final position.



Displacement is Change of Position Vector



From ΔOAB $\quad \Delta\vec{r} = \vec{r}_B - \vec{r}_A$

$$\vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

and

$$\vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

Average velocity $= \dfrac{\text{Displacement}}{\text{Time interval}} \Rightarrow \vec{v}_{av} = \dfrac{\Delta\vec{r}}{\Delta t}$

Average speed $= \dfrac{\text{Distance travelled}}{\text{Time interval}}$

For uniform motion

Average speed = | average velocity | = | instantaneous velocity |

Velocity $\vec{v} = \dfrac{d\vec{r}}{dt} = \dfrac{d}{dt}(x\hat{i} + y\hat{j} + z\hat{k}) = \dfrac{dx}{dt}\hat{i} + \dfrac{dy}{dt}\hat{j} + \dfrac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

Average Acceleration $= \dfrac{\text{Total change in velocity}}{\text{Total time taken}} = \vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$

Acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}) = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

Important Points About 1D Motion

- Distance ≥ | displacement | and Average speed ≥ | average velocity |
- If distance > | displacement | this implies

  (a) atleast at one point in path, velocity is zero.

$$\text{Displacement} \underset{\text{Integration}}{\overset{\text{Differentiation}}{\rightleftarrows}} \text{Velocity} \underset{\text{Integration}}{\overset{\text{Differentiation}}{\rightleftarrows}} \text{Acceleration}$$

Motion with Constant Acceleration: Equations of Motion

- *In vector form:*

$$\vec{v} = \vec{u} + \vec{a}t \text{ and } \Delta\vec{r} = \vec{r}_2 - \vec{r}_1, \vec{s} = \left(\frac{\vec{u}+\vec{v}}{2}\right)t = \vec{u}t + \frac{1}{2}\vec{a}t^2 = \vec{v}t - \frac{1}{2}\vec{a}t^2$$

$$v^2 = u^2 + 2\vec{a}.\vec{s} \text{ and } \vec{s}_{n^{th}} = \vec{u} + \frac{\vec{a}}{2}(2n-1)$$

($S_{n^{th}} \rightarrow$ displacement in n^{th} second)

- *In scalar form (for one dimensional motion):*

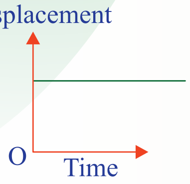
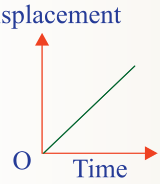
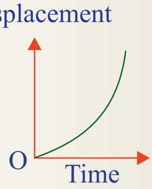
$$v = u + at \qquad s = \left(\frac{u+v}{2}\right)t = ut + \frac{1}{2}at^2 = vt - \frac{1}{2}at^2$$

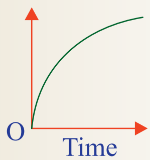
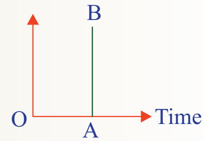
$$v^2 = u^2 + 2as \qquad s_n = u + \frac{a}{2}(2n-1)$$

UNIFORM MOTION

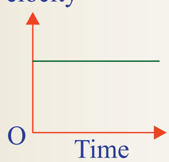
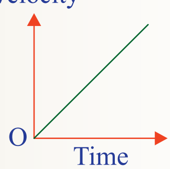
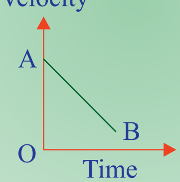
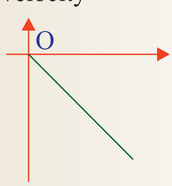
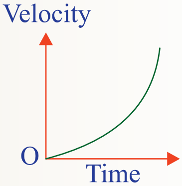
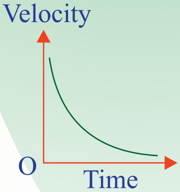
If an object is moving along the straight line covers equal distance in equal interval of time, it is said to be in uniform motion along a straight line.

DIFFERENT GRAPHS OF MOTION DISPLACEMENT-TIME GRAPH

S. No.	Condition	Graph
(a) For a stationary body 	(b) Body moving with a constant velocity 	(c) Body moving with a constant acceleration 

S. No.	Condition	Graph
(d) Body moving with a constant retardation Displacement 	(e) Body moving with infinite velocity. But such motion of body is never possible Displacement 	

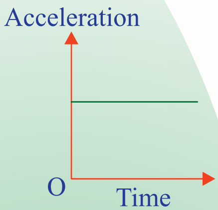
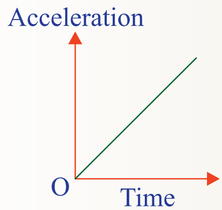
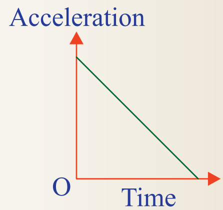
VELOCITY-TIME GRAPH

S.No.	Condition	Graph
(a) Moving with a constant velocity Velocity 	(b) Moving with a constant acceleration having zero initial velocity Velocity 	(c) Body moving with a constant retardation and its initial retar-dation and its initial velocity is zero Velocity 
(d) Moving with a constant retardation with zero initial velocity 	(e) Moving with increasing acceleration Velocity 	(f) Moving with decreasing acceleration Velocity 

NOTES

Slope of velocity-time graph gives acceleration.

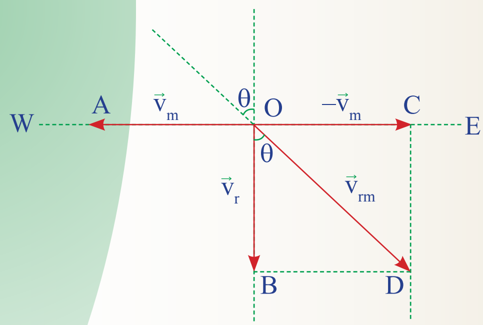
ACCELERATION-TIME GRAPH

S.No.	Condition	Graph
(a) When object is moving with constant acceleration 	(b) When object is moving with constant increasing acceleration 	(c) When object is moving with constant decreasing acceleration 

RELATIVE MOTION

Relative Velocity of Rain w.r.t. the Moving Man

A man walking west with velocity $\vec{v}_m$, represented by $\overrightarrow{OA}$. Let the rain be falling vertically downwards with velocity $\vec{v}_r$, represented by $\overrightarrow{OB}$ as shown in figure.



The relative velocity of rain w.r.t. man $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$ will be represented by diagonal $\overrightarrow{OD}$ of rectangle OBDC.

$$\therefore \qquad v_{rm} = \sqrt{v_r^2 + v_m^2 + 2v_r v_m \cos 90^\circ} = \sqrt{v_r^2 + v_m^2}$$

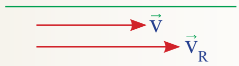
If θ is the angle which $\vec{v}_{rm}$ makes with the vertical direction then

$$\tan\theta = \frac{BD}{OB} = \frac{v_m}{v_r} \Rightarrow \theta = \tan^{-1}\left(\frac{v_m}{v_r}\right)$$

Swimming into the River

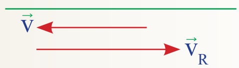
A man can swim with velocity $\vec{v}$, i.e., it is the velocity of man w.r.t. still water. If water is also flowing with velocity $\vec{v}_R$ then velocity of man relative to ground $\vec{v}_m = \vec{v} + \vec{v}_R$.

If the swimming is in the direction of flow of water or along the downstream then



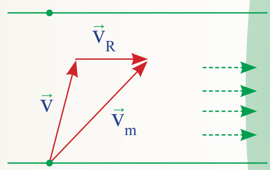
$v_m = v + v_R$

If the swimming is in the direction opposite to the flow of water or along the upstream then

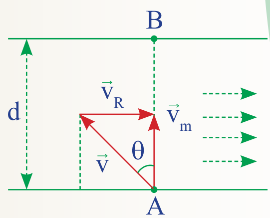


$v_m = v - v_R$

If man is crossing the river as shown in the figure i.e. $\vec{v}$ and $\vec{v}_R$ not collinear then use the vector algebra $\vec{v}_m = \vec{v} + \vec{v}_R$ **(assuming $v > v_R$)**



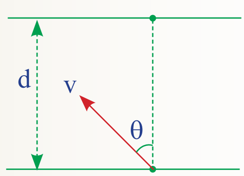
For shortest path



For minimum displacement

To reach at B, $v \sin\theta = v_R \Rightarrow \sin\theta = \dfrac{v_R}{v}$

Time of crossing

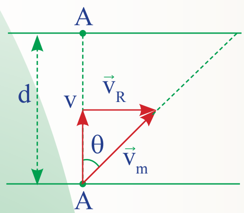


$t = (d/v \cos\theta)$

![NOTES]

If $v_R > v$ then for minimum drifting $\sin\theta = \dfrac{v}{v_R}$.

For minimum time



then $\boxed{t_{min} = \dfrac{d}{v}}$

(For minimum time)

MOTION UNDER GRAVITY (No Air Resistance)

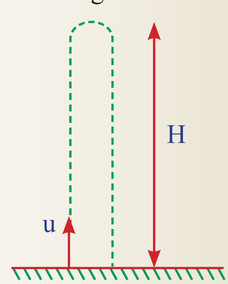
If an object is falling freely (u = 0) under gravity, then equations of motion becomes

(*i*) $v = u + gt$ (*ii*) $h = ut + \dfrac{1}{2}gt^2$ (*iii*) $v^2 = u^2 + 2gh$

NOTES

If an object is thrown upward then g is replaced by –g in above three equations.

If a body is thrown vertically up with a velocity u in the uniform gravitational field then



(*i*) Maximum height attained $H = \dfrac{u^2}{2g}$

(*ii*) Time of ascent = time of descent = $\dfrac{u}{g}$

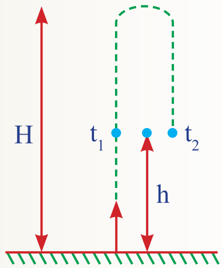
(*iii*) Total time of flight = $\dfrac{2u}{g}$

(*iv*) Velocity of fall at the point of projection = u (downwards)

(v) **Gallileo's law of odd numbers:** For a freely falling body ratio of successive distance covered in equal time interval 't'

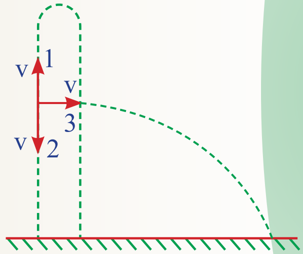
$S_1 : S_2 : S_3 : \ldots, S_n = 1 : 3 : 5 : \ldots, 2n - 1$

At any point on its path the body will have same speed for upward journey and downward journey.



If a body thrown upwards crosses a point in time t_1 and t_2 respectively then height of point $h = \frac{1}{2} g t_1 t_2$. Maximum height $H = \frac{1}{8} g(t_1 + t_2)^2$.

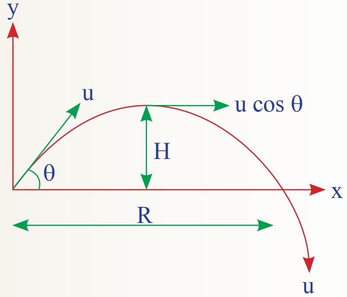
A body is thrown upward, downward and horizontally with same speed takes time t_1, t_2 and t_3 respectively to reach the ground then $t_3 = \sqrt{t_1 t_2}$ and height from where the particle was throw is $H = \frac{1}{2} g t_1 t_2$.



PROJECTION MOTION

Horizontal Motion of Projectile

$$u \cos\theta = u_x$$
$$a_x = 0$$
$$x = u_x t = (u \cos\theta)t$$



Vertical Motion of Projectile

$v_y = u_y - gt$ where $u_y = u \sin\theta$; $y = u_y t - \frac{1}{2}gt^2 = u \sin\theta t - \frac{1}{2}gt^2$

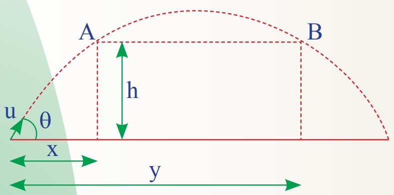
Net acceleration = $\vec{a} = a_x\hat{i} + a_y\hat{j} = -g\hat{j}$

At any Instant

$v_x = u \cos\theta$, $\quad v_y = u \sin\theta - gt$

For Projectile Motion

A body crosses two points at same height in time t_1 and t_2 the points are at distance x and y from starting point then



(a) $x + y = R$

(b) $t_1 + t_2 = T$

(c) $h = \frac{1}{2} g t_1 t_2$

(d) Average velocity from A to B is $u \cos\theta$

If a person can throw a ball to a maximum distance 'x' then the maximum height to which he can throw the ball will be (x/2).

Velocity of Particle at Time t

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = u_x\hat{i} + (u_y - gt)\hat{j} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$$

If angle of velocity $\vec{v}$ from horizontal is α, then

$$\tan\alpha = \frac{v_y}{v_x} = \frac{u_y - gt}{u_x} = \frac{u\sin\theta - gt}{u\cos\theta} = \tan\theta - \frac{gt}{u\cos\theta}$$

At highest point: $v_y = 0,\ v_x = u\cos\theta$

Time of flight: $T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$

Horizontal range: $R = (u\cos\theta)T = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin 2\theta}{g} = \frac{2u_x u_y}{g}$

It is same for q and (90° – θ) and maximum for θ = 45°.

Maximum height: $H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g} = \frac{1}{8}gT^2$

$$\frac{H}{R} = \frac{1}{4}\tan\theta$$

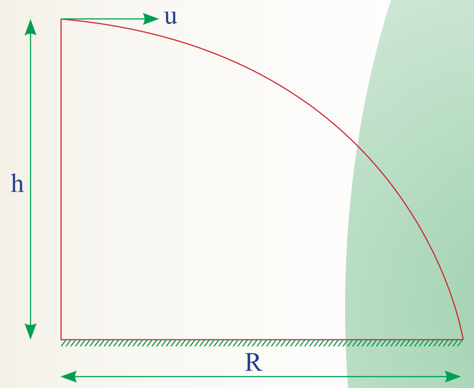
Equation of trajectory $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} = x\tan\theta\left(1 - \frac{x}{R}\right)$

Horizontal Projection from a Height h

Time of flight $T = \sqrt{\frac{2h}{g}}$

Horizontal range $R = uT = u\sqrt{\frac{2h}{g}}$

Angle of velocity at any instant with horizontal $\theta = \tan^{-1}\left(\frac{gt}{u}\right)$



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CHAPTER 5

Newton's Laws of Motion

FROM FIRST LAW OF MOTION

A body continues to be in its state of rest or in uniform motion along a straight line unless an external force is applied on it.

This law is also called law of intertia.

Example: If a moving vehicle suddenly stops, then the passengers inside the vehicle bend outward.

FROM SECOND LAW OF MOTION

$$F_x = \frac{dP_x}{dt} = ma_x \; ; \; F_y = \frac{dP_y}{dt} = ma_y \; ; \; F_z = \frac{dP_z}{dt} = ma_z$$

FROM THIRD LAW OF MOTION

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

$\vec{F}_{AB}$ = Force on A due to B

$\vec{F}_{BA}$ = Force on B due to A

Force

Force is a push or pull which changes or tries to change the state of rest, the state of uniform motion, size or shape of body.

Its SI unit is newton (N) and its dimensional formula is $[MLT^{-2}]$.

Forces can be categorised into two types:

(*i*) **Contact Forces:** Frictional force, tensional force, spring force, normal force etc are the contact forces.

(*ii*) **Distant Forces:** (Field Forces) Electrostatic force, gravitational force, magnetic force etc. are action at a distance forces.

Weight (w)

It is a field force. It is the force with which a body is pulled towards the centre of the earth due to gravity. It has the magnitude mg, where m is the mass of the body and g is the acceleration due to gravity.

$$w = mg$$

Weighing Machine

A weighing machine does not measure the weight but measures the force exerted by object on its upper surface.

Normal Reaction

It is a contact force. It is the force between two surfaces in contact, which is always perpendicular to the surfaces in contact.

Tension

Tension force always pulls a body.

Tension is a reactive force. It is not an active force.

Tension across a massless pulley or frictionless pulley remains constant.

Rope becomes slack when tension force becomes zero.

Spring Force

$$F = -kx$$

x is displacement of the free end from its natural length or deformation of the spring where K = spring constant.

Spring Property

$K \times \ell = \text{constant}$

where ℓ = Natural length of spring

If spring is cut into two in the ratio m : n then spring constant is given by

$$\ell_1 = \frac{m\ell}{m+n}; \ \ell_2 = \frac{n\ell}{m+n}$$

$$k\ell = k_1\ell_1 = k_2\ell_2$$

For series combination of springs

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots$$

For parallel combination of spring

$$k_{eq} = k_1 + k_2 + k_3 \ldots$$

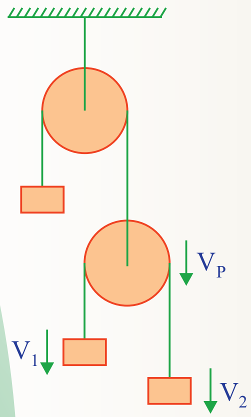
Spring Balance

It does not measure the weight. It measures the force exerted by the object at the hook.

Remember

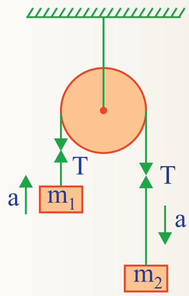
$$V_P = \frac{V_1 + V_2}{2}$$

$$a_P = \frac{a_1 + a_2}{2}$$



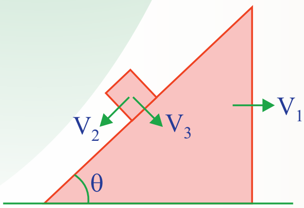
$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

$$T = \frac{2m_1m_2g}{m_1 + m_2}$$

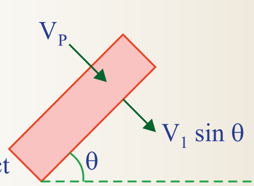


WEDGE CONSTRAINT

Components of velocity along perpendicular direction to the contact plane of the two objects is always equal if there is no deformations at contact place and they remain in contact.



$V_3 = V_1 \sin\theta$



NEWTON'S LAW FOR A SYSTEM

$$\vec{F}_{ext} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots$$

F_{ext} = Net external force on the system.

m_1, m_2, m_3 are the masses of the objects of the system and a_1, a_2, a_3 are the acceleration of the objects respectively.

Equilibrium of a Particle

When the vector sum of the forces acting on a body is zero, then the body is said to be in equilibrium.

$$F_2 \longleftarrow \overset{O}{\bullet} \longrightarrow F_1$$

If two forces F_1 and F_2 act on a particles, then they will be in equilibrium if $F_1 + F_2 = 0$.

Strategy for solving problems in static equilibrium

- Determine *all* the forces that are acting on the rigid body. They will come from the other objects with which the body is in contact (supports, walls, floors, weights resting on them) as well as gravity,
- Draw a diagram and put in *all* the information you have about these forces: The points on the body at which they act, their magnitudes (if known), their directions (if known).
- Write down the equations for static equilibrium. For the torque equation you will have a choice of where to put the axis: in making your choice think of which point would make the resulting equations the simplest.
- Solve the equations! (That's not physics... that's math.) If the problem is well-posed you will not have too many or too few equations to find all the unknowns.

FRICTION

Friction force is of two types–(a) Kinetic, (b) Static.

Kinetic Friction

$$f_k = \mu_k \times N$$

The proportionality constant μ_k is called the coefficient of kinetic friction and its value depends on the nature of the two surfaces in contact.

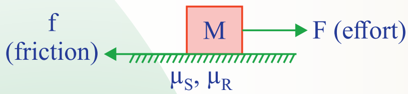
Static Friction

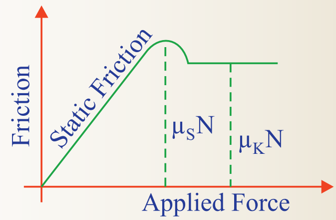
It exists between the two surfaces when there is tendency of relative motion but no relative motion along the two contact surfaces.

This means static friction is a variable and self adjusting force. However it has a maximum value called limiting friction.

$$f_{max} = \mu_S N$$

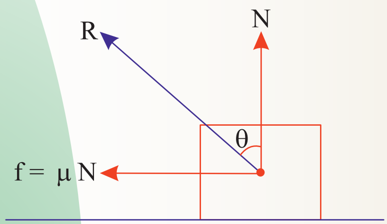
$$0 < f_S < f_{smax}$$





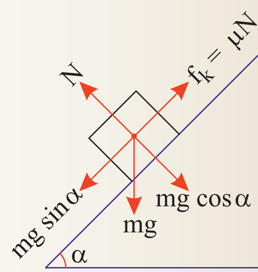
Angle of Friction

It is the angle which the resultant ofo the force of limiting friction and the normal reaction (N) makes with the direction of N.



$$\mu_l = \tan \theta$$

Angle of Repose or Angle of Sliding



It is the minmum angle of inclination of a plane with the horizontal, such that a body placed on it, just begins to slide down.

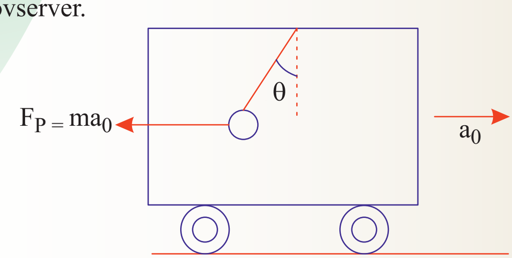
If angle of repose is α and coefficient of limiting friction is μ_l, then

$$\mu_l = \tan \alpha$$

Pseudo Force

When an observer is on an accelerating frame of reference, the observer will measure acceleration on another mass without any external force.

If a_0 is acceleration of observer and he measures the pseudo force F_P on rest mass m, the magnitude of pseudo $F_P = ma_0$ and its direction is opposite to direction of ovserver.



CIRCULAR MOTION

Definition of Circular Motion

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as circular motion with respect to that fixed point. That fixed point is called centre and the distance is called radius of circular path.

Radius Vector

The vector joining the centre of the circle and the centre of the particle performing circular motion is called radius vector. It has constant magnitude and variable direction. It is directed outwards.

Frequency (n or f)

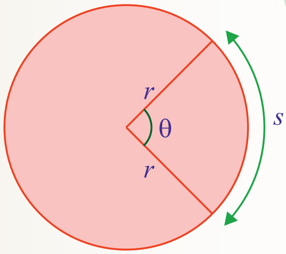
Number of revolutions described by particle per sec. Its unit is revolutions per second (r.p.s.) or revolutions per minute (r.p.m.).

Time Period (T)

It is time taken by particle to complete one revolution.

$$T = \frac{1}{n}$$

- Angle $\theta = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$ (Unit $\rightarrow$ radian)
- Average angular velocity $\omega = \frac{\Delta\theta}{\Delta t}$ (a scalar) unit $\rightarrow$ rad/sec
- Instantaneous angular velocity $\omega = \frac{d\theta}{dt}$ (a vector) unit $\rightarrow$ rad/sec



- For uniform angular velocity $\omega = \frac{2\pi}{T} = 2\pi f$ or $2\pi n$
- Angular displacement $\theta = \omega t$

- Relation between ω (uniform) and v $\omega = \frac{v}{r}$

- In vector from velocity $\vec{v} = \vec{\omega} \times \vec{r}$

- Acceleration $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r}) = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$

$$= \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v} = \vec{a}_t + \vec{a}_C$$

- Tangential acceleration: $a_t = \frac{dv}{dt} = \alpha r$

- Centripetal acceleration : $a_C = \omega v = \frac{v^2}{r} = \omega^2 r$ or $\vec{a}_C = \omega^2 r(\hat{r})$

- Magnitude of net acceleration : $a = \sqrt{a_C^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$

Maximum/Minimum Speed in Circular Motion

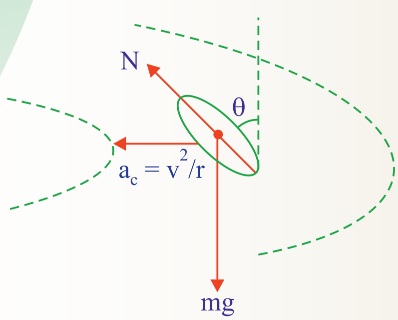
- On unbanked road : $v_{max} = \sqrt{\mu_s Rg}$

- On banked road : $v_{max} = \sqrt{\left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)Rg}$

$$v_{min} = \sqrt{\frac{(\tan\theta - \mu_S)Rg}{1 + \mu_S \tan\theta}} \qquad v_{min} \le v_{car} \le v_{max}$$

where ϕ = angle of friction = $\tan^{-1}\mu_s$; θ = angle of banking.

- Bending of cyclist : $\tan\theta = \frac{v^2}{rg}$



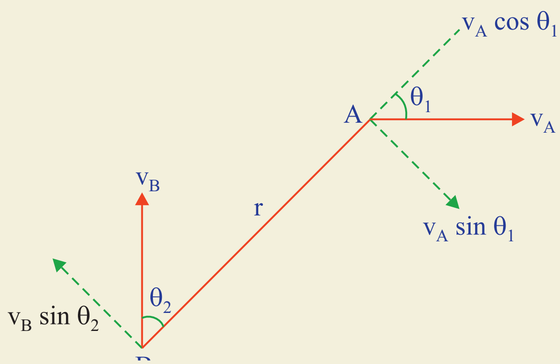
KEY TIPS

- Average angular velocity is a scalar physical quantity whereas instantaneous angular velocity is a vector physical quantity.
- Small Angular displacement $d\vec{\theta}$ is a vector quantity, but large angular displacement θ is scalar quantity.

$$d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1 \quad \text{But } \vec{\theta}_1 + \vec{\theta}_2 \neq \vec{\theta}_2 + \vec{\theta}_1$$

Relative Angular Velocity

Relative angular velocity of a particle 'A' w.r.t. other moving particle B is the angular velocity of the position vector of A w.r.t. B.



That means it is the rate at which position vector of 'A' w.r.t. B rotates at that instant

$$\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{\text{Relative velocity of A w.r.t. B perpendicular to line AB}}{\text{separation between A and B}}$$

here $(v_{AB})_\perp = v_A \sin\theta_1 + v_B \sin\theta_2$

$$\therefore \quad \omega_{AB} = \frac{v_A \sin\theta_1 + v_B \sin\theta_2}{r}$$

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CHAPTER 6

Work, Power and Energy

WORK DONE BY CONSTANT FORCE

$$W = \vec{F} \cdot \vec{S}$$

Work Done by Multiple Forces

$$\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

$$W = [\Sigma\vec{F}] \cdot \vec{S} \qquad \ldots(i)$$

$$W = \vec{F}_1 \cdot \vec{S} + \vec{F}_2 \cdot \vec{S} + \vec{F}_3 \cdot \vec{S} + \cdots$$

or $$W = W_1 + W_2 + W_3 + \ldots$$

Work Done by Variable Force

$$W = \int dW = \int \vec{F} \cdot d\vec{s}$$

Area under the force and displacement curve gives work done.

RELATION BETWEEN MOMENTUM AND KINETIC ENERGY

$$K = \frac{p^2}{2m} \text{ and } P = \sqrt{2mK}\,;\ P = \text{linear momentum}$$

Potential Energy

$$\int_{U_1}^{U_2} dU = -\int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$

i.e., $$U_f - U_i = -\int_{r_i}^{r_f} \vec{F} \cdot d\vec{r} = -W_{\text{Conservative}}$$

Conservative Forces

$$\vec{F} = -\frac{dU}{dr}\hat{r} \qquad \vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

EQUILIBRIUM CONDITIONS

Stable Equilibrium

$$F(r) = -\frac{\partial U}{\partial r} = 0; \quad \text{and} \quad \frac{\partial F}{\partial r} < 0; \quad \text{and} \quad \frac{\partial^2 U}{\partial r^2} > 0$$

Unstable Equilibrium

$$F(r) = -\frac{\partial U}{\partial r} = 0; \quad \text{therefore} \quad \frac{\partial F}{\partial r} > 0; \quad \text{and} \quad \frac{\partial^2 U}{\partial r^2} < 0$$

Neutral Equilibrium

$$F(r) = -\frac{\partial U}{\partial r} = 0; \quad \text{therefore} \quad \frac{\partial F}{\partial r} = 0; \quad \text{and} \quad \frac{\partial^2 U}{\partial r^2} = 0$$

WORK-ENERGY THEOREM

$$W_{All} = \Delta K$$

$$\Rightarrow \qquad W_C + W_{NC} + W_{Ext} + W_{Pseudo} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

LAW OF CONSERVATION OF MECHANICAL ENERGY

If the net external force acting on a system is zero, then the mechanical energy is conserved.

$$K_f + U_f = K_i + U_i$$

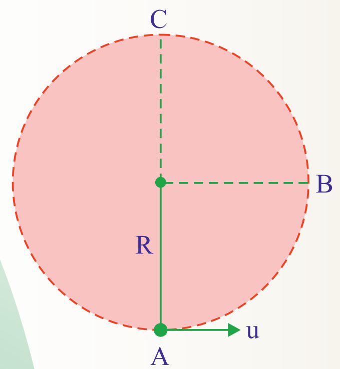
POWER

The average poiwer delivered by an agent is given by

$$P_{avg} = \frac{W}{t}$$

$$P = \frac{d\vec{F}\cdot\vec{S}}{dt} = \vec{F}\cdot\frac{d\vec{S}}{dt} = \vec{F}\cdot\vec{v}$$

Circular Motion in Vertical Plane



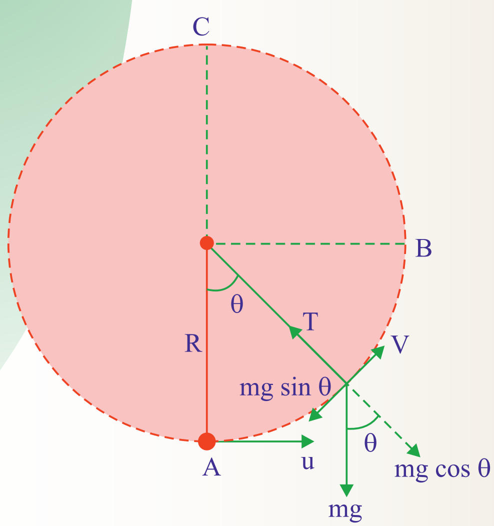
A. Condition to completge vertical circle $u \geq \sqrt{5gR}$

If $u = \sqrt{5gR}$ then Tension at C is equal to 0 and tension at A is equal to 6 mg.

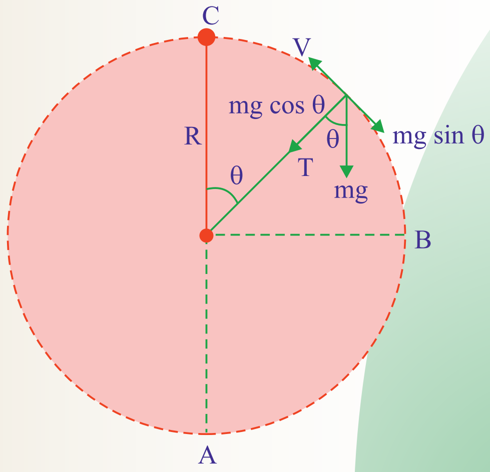
Velocity at B : $v_B = \sqrt{3gR}$

Velocity at C : $v_C = \sqrt{gR}$

From A to B : $T = mg\cos\theta + \dfrac{mv^2}{R}$



From B to C : $T = \dfrac{mv^2}{R} - mg\cos\theta$



B. Condition for pendulum motion (oscillating condition)

$$u \le \sqrt{2gR} \text{ (in between A to B)}$$

Velocity can be zero but T never be zero between A and B.

Because T is given by $T = mg\cos\theta + \dfrac{mv^2}{R}$.

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CHAPTER 7

Centre of Mass and Collision

CENTRE OF MASS OF A SYSTEM OF 'N' DISCRETE PARTICLES

$$r_{cm} = \frac{m_1 r_1 + m_2 r_2 + \cdots + m_n r_n}{m_1 + m_2 + \cdots + m_n} \; ; \; r_{cm} = \frac{\sum_{i=1}^{n} m_i r_i}{\sum_{i=1}^{n} m_i} \quad r_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i r_i$$

CENTRE OF MASS OF A CONTINUOUS MASS DISTRIBUTION

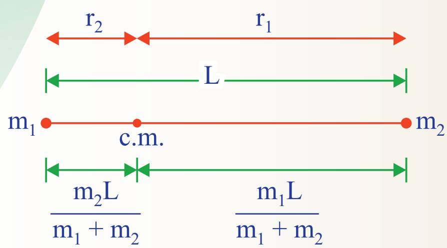
$$x_{cm} = \frac{\int x\,dm}{\int dm}, \quad y_{cm} = \frac{\int y\,dm}{\int dm}, \quad z_{cm} = \frac{\int z\,dm}{\int dm}$$

$$\int dm = M(\text{mass of the body})$$

CENTRE OF MASS OF SOME COMMON SYSTEMS

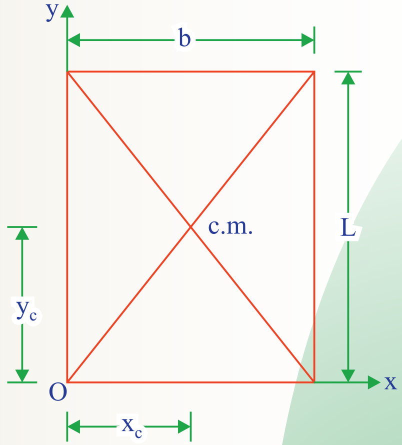
A system of two point masses $m_1 r_1 = m_2 r_2$.

The centre of mass lies closer to the heavier mass.



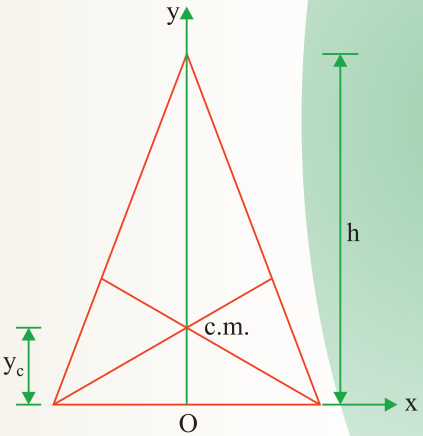
Rectangular plate

$$x_c = \frac{b}{2} \qquad y_c = \frac{L}{2}$$



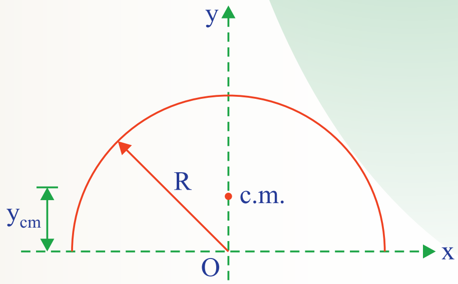
A triangular plate at the centroid

$$y_c = \frac{h}{3}$$



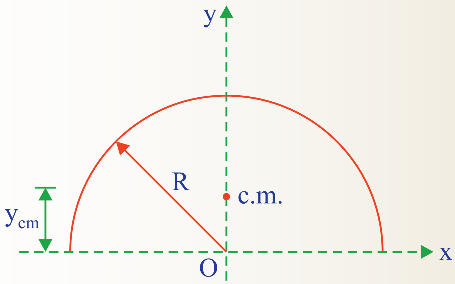
A semi-circular ring

$$y_c = \frac{2R}{\pi} \qquad x_c = O$$



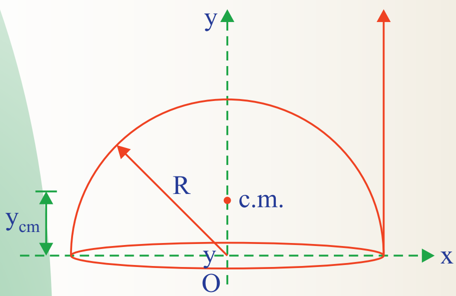
A semi-circular disc

$$y_c = \frac{4R}{3\pi} \qquad x_a = O$$



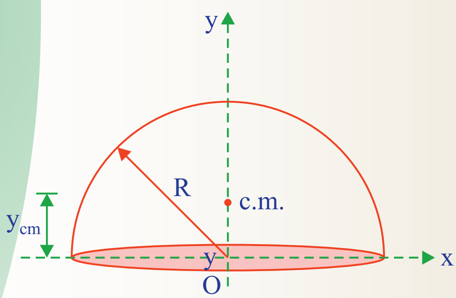
A hemispherical shell

$$y_c = \frac{R}{2} \qquad x_c = O$$



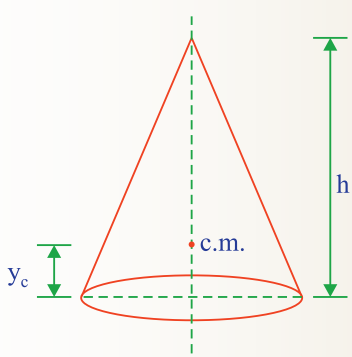
A solid hemisphere

$$y_c = \frac{3R}{8} \qquad x_c = O$$

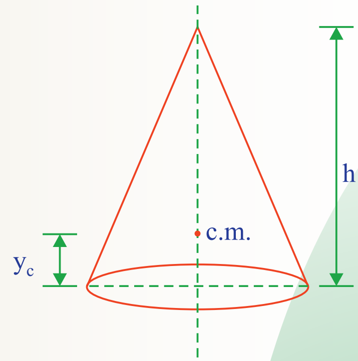


A circular cone (solid)

$$y_c = \frac{h}{4}$$



A circular cone (hollow)



$$y_c = \frac{h}{3}$$

MOTION OF CENTRE OF MASS AND CONSERVATION OF MOMENTUM

Velocity of Centre of Mass of System

$$\vec{v}_{cm} = \frac{m_1\frac{\overrightarrow{dv_1}}{dt} + m_2\frac{\overrightarrow{dr_2}}{dt} + m_3\frac{\overrightarrow{dr_3}}{dt}\cdots + m_n\frac{\overrightarrow{dr_n}}{dt}}{M} = \frac{m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2} + m_3\overrightarrow{v_3}\ldots + m_n\overrightarrow{v_n}}{M}$$

$$\vec{P}_{sys} = M\vec{v}_{cm}$$

Acceleration of Centre of Mass of System

$$\vec{a}_{cm} = \frac{m_1\frac{\overrightarrow{dV_1}}{dt} + m_2\frac{\overrightarrow{dV_2}}{dt} + m_3\frac{\overrightarrow{dV_3}}{dt}\cdots + m_n\frac{\overrightarrow{dV_n}}{dt}}{M}$$

$$= \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3\ldots + m_n\vec{a}_n}{M}$$

$$= \frac{\text{Net force on system}}{M} = \frac{\text{Net external force + Net internal force}}{M}$$

$$= \frac{\text{Net External Force}}{M} \quad (\because \quad \Sigma\ \text{Internal force} = 0)$$

$$\vec{F}_{ext} = M\vec{a}_{cm}$$

Impulse of a force F on a body is defined as:

$$J = \int_{t_i}^{t_f}\vec{F}\,dt = \int_{t_i}^{t_f}d\vec{P} = \Delta P \qquad \text{(Area under the Force vs time curve gives the impulse)}$$

$$\vec{J} = \Delta\vec{P} \quad \textbf{(impulse – momentum theorem)}$$

Principle of Conservation of linear momentum

- If, $\left(\sum F_{ext}\right)_{system} = 0 \Rightarrow (P_i)_{system} = (P_f)_{system}$
- $(KE)_{system} = \frac{1}{2}(m_1 v_1^2 + m_2 v_2^2 + ... m_n v_n^2) \neq \frac{1}{2} M V_{com}^2$

COEFFICIENT OF RESTITUTION (e)

Before collision After collision

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$e = \frac{\text{Impulse of reformation}}{\text{Impulse of deformation}} = \frac{\int F_r dt}{\int F_d dt}$$

$$= \frac{\text{Velocity of separation along line of impact}}{\text{Velocity of approach along line of impact}}$$

$$V_1 = \frac{P_i + m_2 e(u_2 - u_1)}{m_1 + m_2}, \qquad V_2 = \frac{P_i + m_1 e(u_1 - u_2)}{m_1 + m_2}$$

(a) $e = 1$ Impulse of Reformation = Impulse of Deformation
Velocity of separation = Velocity of approach
Kinetic Energy may be conserved
Elastic collision.

(b) $e = 0$ Impulse of Reformation = 0
Velocity of separation = 0
Kinetic Energy is not conserved
Perfectly Inelastic collision.

(c) $0 < e < 1$ Impulse of Reformation < Impulse of Deformation
Velocity of seapration < Velocity of approach
Kinetic Energy is not conserved
Inelastic collision.

VARIABLE MASS SYSTEM

If a mass is added or ejected from a system, at rate μ kg/s and relative velocity v_{rel} (w.r.t. the system), then the force exerted by this mass on the system has magnitude $\mu\,|v_{rel}|$.

Thrust Force (F_t)

$$F_t = v_{rel}\frac{dm}{dt}$$

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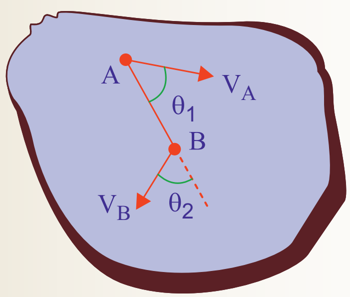
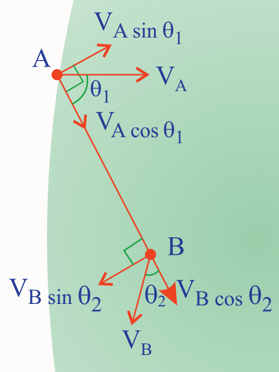
CHAPTER 8

Rigid Body Dynamics

RIGID BODY

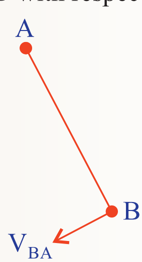




If the above body is rigid

$$V_A \cos\theta_1 = V_B \cos\theta_2$$

V_{BA} = relative velocity of point B with respect to point A.



Types of Motion of rigid body

- Pure Translational Motion
- Pure Rotational Motion
- Combined Translationaland Rotational Motion

MOMENT OF INERTIA (I)

Definition: Moment of Inertia is defined as the capability of system to oppose the change produced in the rotational motion of a body.

Moment of Inertia is a scalar (positive quantity).

$$I = m_1 r_1^2 + m_2 r_2^2 + \ldots$$

$$= I_1 + I_2 + I_3 + \ldots$$

SI unit of Moment of Inertia is Kgm^2.

Moment of Inertia of

A single particle

$$I = mr^2$$

where m = mass of the particle

r = perpendicular distance of the particle from the axis about which moment of Inertia is to be calculated

For a continuous object

$$I = \int dI = r^2 \int dm$$

where, dI = moment of inertia of a small element

dm = mass of a small element

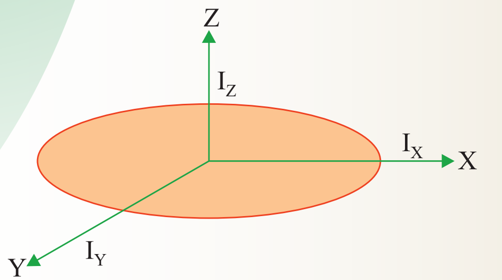
r = perpendicular distance of the particle from the axis

TWO IMPORTANT THEOREMS ON MOMENT OF INERTIA

Perpendicular Axis Theorem

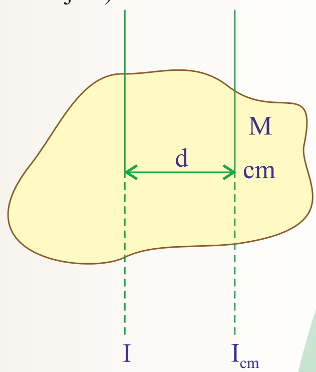
Only applicable to plane lamina (that means for 2-D objects only)

$$I_Z = I_X + I_Y \text{ (when object is in } x - y \text{ plane).}$$



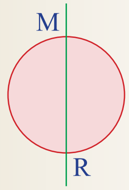
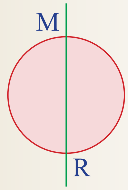
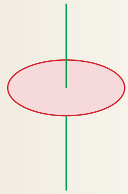
Parallel Axis Theorem

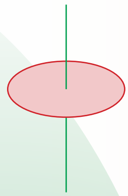
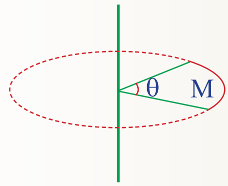
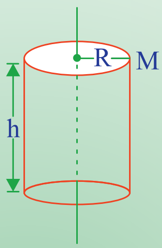
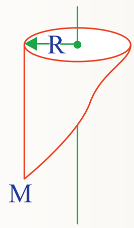
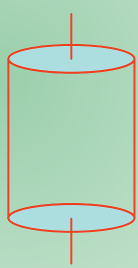
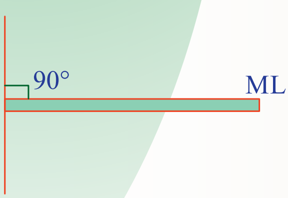
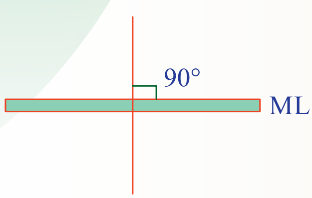
(Applicable to any type of object) :

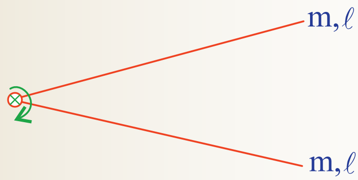


$$I = I_{cm} + Md^2$$

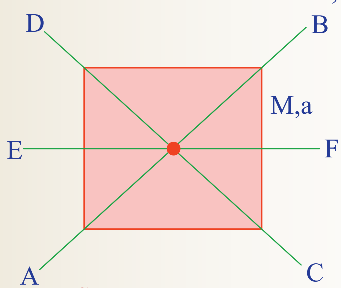
List of some useful formula:

Object	Moment of Inertia
 **Solid Sphere**	$\frac{2}{5}MR^2$ (Uniform)
 **Hollow Sphere**	$\frac{2}{3}MR^2$ (Uniform)
 **Ring**	MR^2 (Uniform or Non Uniform)

Object	Moment of Inertia
 **Disc** or  **Sector of Disc**	$\frac{MR^2}{2}$
 **Hollow cylinder** or  **Part of hollow cylinder**	MR^2 (uniform or non uniform)
 **Solid cylinder**	$\frac{MR^2}{2}$ (Uniform)
	$\frac{ML^2}{3}$ (Uniform)
	$\frac{ML^2}{12}$ (Uniform)

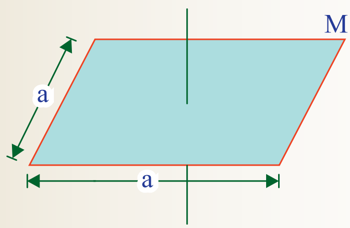


$(2M\ell^2)/3$ (Uniform)



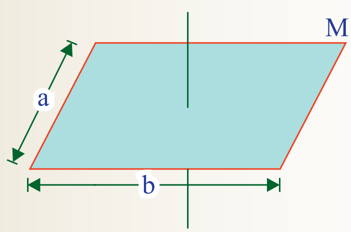
Square Plate

$$I_{AB} = I_{CD} = I_{EF} = \frac{Ma^2}{12} \text{ (Uniform)}$$



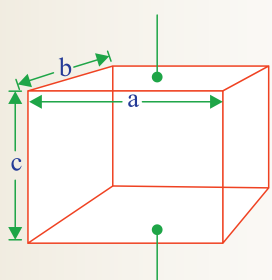
Square Plate

$$\frac{Ma^2}{6} \text{(Uniform)}$$



Rectangular Plate

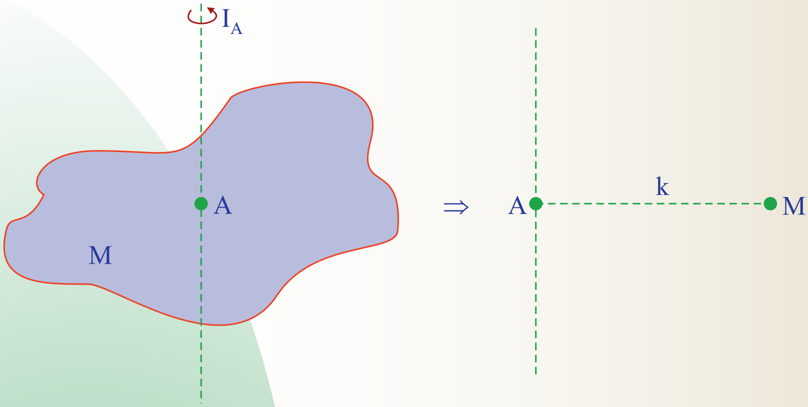
$I = [M(a^2 + b^2)]/12$(Uniform)



Cuboid

$$\frac{M\left(a^2+b^2\right)}{12} \text{ (Uniform)}$$

RADIUS OF GYRATION (k)

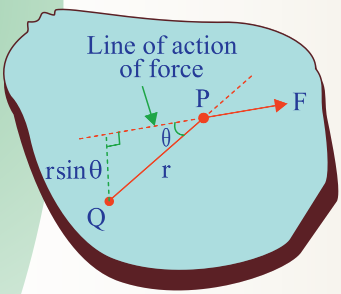


$$I_A = Mk^2$$

$$k = \sqrt{\frac{I_A}{M}}$$

TORQUE

$$\vec{\tau} = \vec{r} \times \vec{F}$$



$$\vec{P} = M\vec{v}_{CM} \qquad \vec{F}_{external} = M\vec{a}_{CM}$$

net external force acting on the body has two perts tangential and centripetal.

$$F_C = ma_C = m\frac{v^2}{r_{CM}} = m\omega^2 r_{CM} \qquad F_t = ma_t = m\alpha\, r_{CM}$$

ROTATIONAL EQUILIBRIUM

For transitional equilibrium.

$$\sum F_x = 0$$

and $$\sum F_y = 0$$

The condition of rotational equilibrium is

$$\sum \vec{\tau} = 0$$

ANGULAR MOMENTUM ($\vec{L}$)

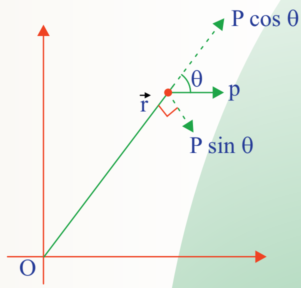
Angular Momentum of a Particle about a Point

$$\vec{L} = \vec{r} \times \vec{P}$$

$$L = rP \sin\theta$$

$$|L| = r_{\perp} \times P$$

$$|L| = P_{\perp} \times r$$



Angular momentum of a rigid body rotating about fixed axis :

$L_H = I_H \omega$

L_H = angular momentum of object about axis H.

I_H = Moment ofi Inertia of rigid object about axis H.

ω = angular velocity of the object.

Conservation of Angular Momentum

Angular momentum of a particle or a system remains constant if $\tau_{ext} = 0$ about that point or axis of rotation.

$$L_i = L_f \Rightarrow I_i\omega_i = I_f\omega_f$$

Relation between Torque and Angular Momentum

$$\tau = \frac{dL}{dt}$$

Torque is change in angular momentum

IMPULSE OF TORQUE

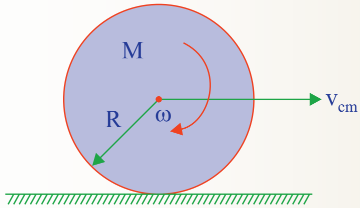
$$\int \tau dt = \Delta J$$

ΔJ = Change in angular momentum.

Rolling Motion

Total kinetic energy $= \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{cm}\omega^2$

Total angular momentum $= Mv_{CM}R + I_{cm}\omega$



Pure Rolling (or Rolling without Slipping) on Stationary Surface

Condition : $v_{cm} = R\omega$

In accelerated motion $a_{cm} = R\alpha$

If $v_{cm} > R\omega$ then rolling with forward slipping.

If $v_{cm} < R\omega$ then rolling with backward slipping.

Total kinetic energy in pure rolling

$$K_{total} = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}(Mk^2)\left(\frac{v_{cm}^2}{R^2}\right) = \frac{1}{2}Mv_{cm}^2\left(1+\frac{k^2}{R^2}\right)$$

Dynamics :

$$\vec{\tau}_{cm} = I_{cm}\vec{\alpha},\ \ \vec{F}_{ext} = M\vec{a}_{cm},\ \ \vec{P}_{system} = M\vec{v}_{cm}$$

Total K.E. $= \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$

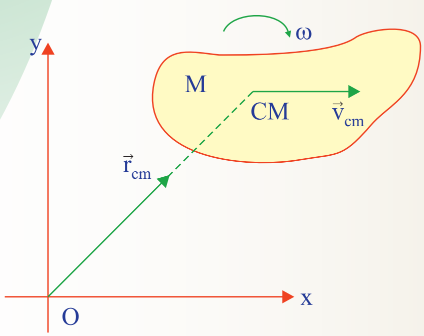
Pure rolling motion on an inclined plane

Acceleration $a = \dfrac{g\sin\theta}{1+k^2/R^2}$

Minimum frictional coefficient $\mu_{min} = \dfrac{\tan\theta}{1+R^2/k^2}$

Angular momentum about axis O = $\vec{L}$ about C.M. + $\vec{L}$ of C.M. about O

$$\vec{L}_O = I_{CM}\vec{\omega} + \vec{r}_{CM} \times M\vec{v}_{CM}$$



CHAPTER 9

Gravitation

NEWTON'S UNIVERSAL LAW OF GRAVITATION

Force of attraction between two point masses $F = \dfrac{Gm_1m_2}{r^2}$,

where $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

Directed along the line joining the point masses.

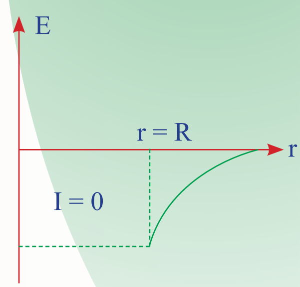
- It is a conservative force $\Rightarrow$ mechanical energy will be conserved.
- It is a central force $\Rightarrow$ angular momentum will be conserved.

Gravitational Field due to Spherical Shell

Outside Region $E_g = \dfrac{GM}{r^2}$, where $r > R$

On the surface $E_g = \dfrac{GM}{R^2}$, where $r = R$

Inside Region $E_g = 0$, where $r < R$



NOTES

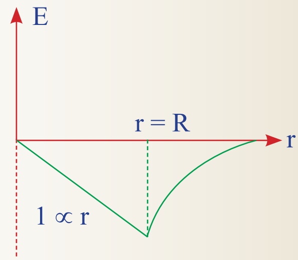
Direction always towards the centre of the sphere, radially inwards.

Gravitational Field due to Solid Sphere

Outside Region $E_g = \dfrac{GM}{r^2}$, where $r > R$

On the surface $E_g = \frac{GM}{R^2}$, where $r = R$

Inside Region $E_g = \frac{GMr}{R^3}$, where $r < R$



Acceleration due to gravity $g_S = \frac{GM}{R^2}$ (on the surface of earth)

At height h, $g_h = \frac{GM}{(R+h)^2}$

If $h << R$; $g_h \approx g_s\left(1 - \frac{2h}{R}\right)$

At depth d, $g_d = \frac{GM(R-d)}{R^3} = g_s\left(1 - \frac{d}{R}\right)$

Effect of rotation on g : $g' = g - \omega^2 R\cos^2\lambda$ where λ is angle of latitude.

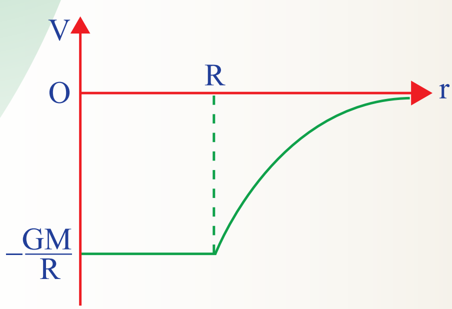
Gravitational potential

Due to a point mass at a distance r is $V = \frac{GM}{r}$

Gravitational potential due to spherical shell

Outside the shell $V = \frac{GM}{r}$, $r > R$

Inside/on the surface of the shell $V = \frac{GM}{R}$, $r < R$

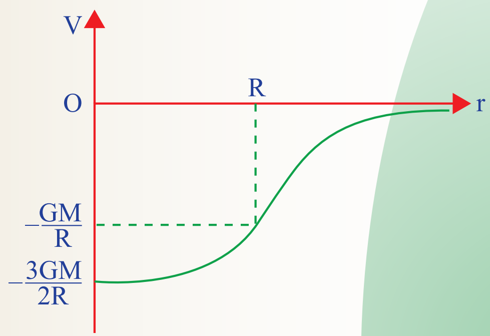


Potential due to a solid sphere

Outside Region $$V = -\frac{GM}{r}, \; r > R$$

On the surface $$V = -\frac{GM}{R}, \; r = R$$

Inside Region $$V = -\frac{GM(3R^2 - r^2)}{2R^3}, \; r < R$$



Potential on the axis of a thin ring at a distance x from the centre $$V = -\frac{GM}{\sqrt{R^2 + x^2}}$$

Escape velocity from a planet of mass M and radius R $$V_e = \sqrt{\frac{2GM}{R}}$$

Orbital velocity of satellite (orbital radius r) $$V_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R+H)}}$$

For nearby satellite $$V_0 = \sqrt{\frac{GM}{R}} = \frac{V_e}{\sqrt{2}}$$

Here V_e = escape velocity on earth surface.

Time Period of Satellite

$$T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

Energies of a Satellite

Potential energy $U = -\dfrac{GMm}{r}$

Kinetic energy $K = \dfrac{1}{2}mv^2 = \dfrac{GMm}{2r}$

Mechanical energy $E = U + K = -\dfrac{GMm}{2r}$

Binding energy $BE = -E = \dfrac{GMm}{2r}$

Kepler's laws

- 1^{st} Law of orbitals Path of a planet is elliptical with the sun at one of the focus.
- II^{nd} Law of areas Areal velocity $\dfrac{d\vec{A}}{dt} = \text{constant} = \dfrac{\vec{L}}{2m}$
- III^{rd} - Law of periods: $T^2 \propto a^3$ or $T^2 \propto \left(\dfrac{r_{max} + r_{min}}{2}\right)^3 \propto (\text{mean radius})^3$ For circular orbits $T^2 \propto R^3$

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Chapter 10

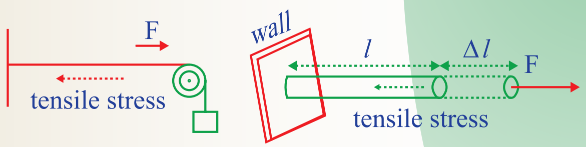
Properties of Matter

$$\textbf{STRESS} = \frac{\text{Internal restoring force}}{\text{Area of cross} - \text{section}} = \frac{F_{Res}}{A}.$$

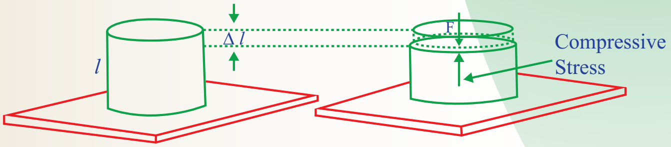
There are three types of stress :-

Longitudinal Stress (2 Types)

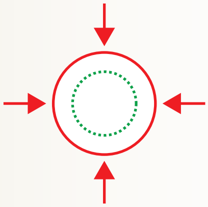
(a) Tensile Stress :



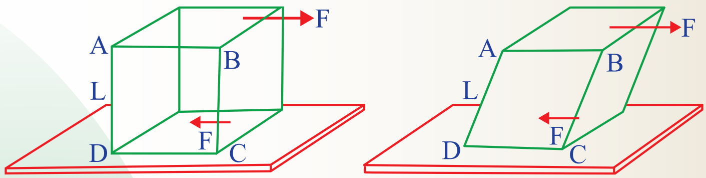
(b) Compressive Stress :



Volume Stress



Tangential Stress or Shear Stress

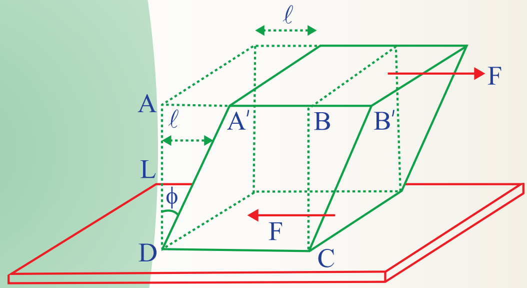


Strain $= \dfrac{\text{Change in Size of the body}}{\text{Original size of the body}}$ (3 types)

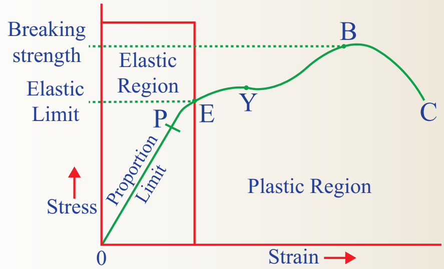
1. Longitudinal strain $= \dfrac{\text{Change in length of the body}}{\text{initial length of the body}} = \dfrac{\Delta L}{L}$

2. Volume strain $= \dfrac{\text{Change in volume of the body}}{\text{original volume of the body}} = \dfrac{\Delta V}{V}$

3. Shear strain : $\tan\phi = \dfrac{\ell}{L}$ or $\phi = \dfrac{\ell}{L} = \dfrac{\text{displacement of upper face}}{\text{distance between two faces}}$



Stress – Strain Curve



Hooke's Law $\boxed{\text{Stress } \alpha \text{ strain}}$ **(within limit of elasticity)**

Young's modulus of elasticity $Y = \dfrac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \dfrac{FL}{A\,\Delta\ell}$

If L is the length of wire, r is radius and ℓ is the increase in length of the wire by suspending a weight Mg at its one end, then Young's modulus of elasticity of the material of wire $Y = \dfrac{(Mg/\pi r^2)}{(\ell/L)} = \dfrac{MgL}{\pi r^2 \ell}$

Increment in Length due to Own Weight

$$\Delta\ell = \frac{MgL}{2AY} = \frac{\rho g L^2}{2Y}$$

$$K = \frac{\text{Volumetric stress}}{\text{Volumetric strain}} = \frac{F/A}{\left(\frac{-\Delta V}{V}\right)} = \frac{P}{\left(\frac{-\Delta V}{V}\right)}$$

Bulk Modulus of Elasticity

$$k = \frac{\text{Volume stress}}{\text{Volume strain}} = \frac{\frac{F}{A}}{-\frac{\Delta V}{V}} = \frac{P}{-\frac{\Delta V}{V}}$$

Compressibility

$$C = \frac{1}{\text{Bulk modulus}} = \frac{1}{K}$$

Modulus of rigidity

$$\eta = \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{(F_{\text{tangential}})/A}{\phi}$$

Poisson's ratio

$$(\sigma) = \frac{\text{lateral strain}}{\text{Longitudinal strain}}$$

Work done in stretching wire

$$W = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}:$$

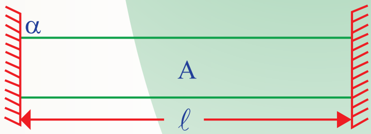
$$W = \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta\ell}{\ell} \times A \times \ell = \frac{1}{2} F \times \Delta\ell$$

Rod is rigidly fixed at the ends, between walls

Thermal strain = $\alpha\Delta\theta$

Thermal stress = Y $\alpha\Delta\theta$

Thermal tension = Y $\alpha A\Delta\theta$



Effect of Temperature on elasticity

When temperature is increased then due to weakness of inter molecular force the elastic properties in general decrease i.e, elastic constant decreases. Plastic properties increase with temperature.

CHAPTER 11

Fluid Mechanics

DENSITY $= \dfrac{\text{mass}}{\text{volume}}$ **($kg\ m^{-3}$)**

Specific weight $= \dfrac{\text{weight}}{\text{volume}} = \rho g$ **($kg\ m^{-2}\ s^{-2}$)**

Relative density $= \dfrac{\text{density of given liquid}}{\text{density of pure water at } 4^{o}C}$ **(Unitless)**

Pressure $= \dfrac{\text{normal force}}{\text{area}} = \dfrac{\text{thrust}}{\text{Area}}$ **(Nm^{-2})**

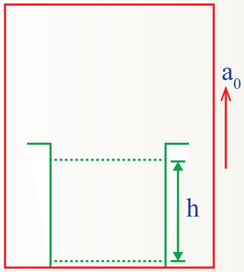
Variation of pressure with depth

Pressure is same at two points at the same horizontal level P_1 and P_2 The difference of pressure between two points separated by a depth h is

$$(P_2 - P_1) = h\rho g$$

Pressure in case of accelerating fluid

Liquid placed in elevator: When elevator accelerates upward with acceleration a_0 then pressure in the fluid, at depth h may be given by, $P = h\rho[g + a_0]$



Steady and Unsteady Flow : Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure and density at a point do not change with time.

Streamline Flow : In steady flow all the particles passing through a given point follow the same path and hence a unique line of flow. This line or path is called a streamline.

Laminar and Turbulent Flow : Laminar flow is the flow in which the fluid particles move along well-defined streamlines which are straight and parallel.

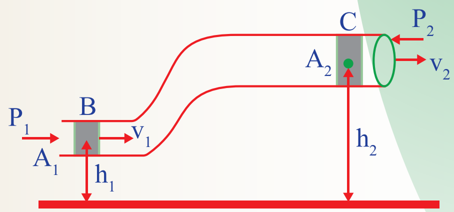
Compressible and Incompressible Flow : In compressible flow the density of fluid varies from point to point i.e, the density is not constant for the fluid whereas in incompressible flow the density of the fluid remains constant throughout.

Rotational and Irrotational Flow : Rotational flow is the flow in which the fluid particles while flowing along path-lines also rotate about their own axis, In irrotational flow particles do not rotate about their axis.

Equation of continuity $A_1v_1 = A_2v_2$ Based on conservation of mass.

Bernoulli's theorem : $\boxed{P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}}$

Based on the conservation of energy.



$$P_1 + \rho gh_1 + \frac{\rho v_1^2}{2} = P_2 + \rho gh_2 + \frac{\rho v_2^2}{2}$$

Kinetic Energy

Kinetic energy per unit volume $= \dfrac{\text{Kinetic Energy}}{\text{volume}} = \dfrac{1}{2}\dfrac{m}{V}v^2 = \dfrac{1}{2}\rho v^2$

Potential Energy

Potential energy per unit volume $= \dfrac{\text{Potential Energy}}{\text{volume}} = \dfrac{m}{V}gh = \rho gh$

Pressure Energy

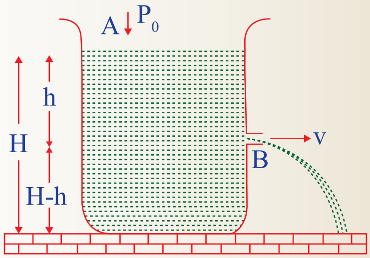
Pressure energy per unit volume $= \dfrac{\text{Pressure energy}}{\text{volume}} = P$

Rate of flow :

Volume of water flowing per second $Q = A_1v_1 = A_2v_2$

Velocity of efflux $V = \sqrt{2gh}$

Horizontal range R $= 2\sqrt{h(H-h)}$



SURFACE TENSION

Surface tension is basically a property of liquid. The liquid surface behaves like a stretched elastic membrane which has a natural tendency to contract and tends to have a minimum surface area. This property of liquid is called surface tension. It arises due to intermolecular forces in a liquid.

Intermolecular forces

(a) Cohesive Force

The force acting between the molecules of same substance is called cohesive force.

(b) Adhesive Force

The force acting between different types of molecules or molecules of different substances is called adhesive force.

- Intermolecular forces are different from the gravitational forces and do not obey the inverse-square law
- The distance upto which these forces are effective, is called-molecular range, This distance is nearly 10^{-9} m. Within this limit this increase very rapidly as the distance decrease.
- Molecular range depends on the nature of the substance.

Properties of surface tension

- Surface tension is a scalar quantity.
- It acts tangential to liquid surface.
- Surface tension is always produced due to cohesive force.
- More is the cohesive force, more is the surface tension.
- When surface area of liquid is increased, molecules from the interior of the liquid rise to the surface. For this, work is done against the downward cohesive force.

Dependency of surface Tension

On Cohesive Force : Those factors which increase the cohesive force between molecules increase the surface tension and those which decrease the cohesive force between molecules decrease the surface tension.

(a) On mixing detergent in water its surface tension decreases.

(b) Surface tension of water is more than (alcohol + water) mixture.

On Temperature

On increasing temperature surface tension decreases. At critical temperature and boiling point it becomes zero.

NOTES

Surface tension of water is maximum at 4ºC

On Contamination

The dust particles on the liquid surface decreases its surface tension.

Definition of surface tension

The force acting per unit length of an imaginary line drawn on the free liquid surface at right angles to the line and in the plane of liquid surface, is difined as surface tension.

For floating needle $2T\ell \sin\theta = mg$

Work = Surface energy = $T\Delta A$

Liquid drop $W = 4\pi r^2 T$

Soap bubble $W = 8\pi r^2 T$

Splitting of bigger drop into smaller droplets $R = n^{1/3} r$

Work done = Change in surface energy = $4\pi R^2 T (n^{1/3} - 1)$

Excess pressure $P_{ex} = P_{in} - P_{out}$

In liquid drop $P_{ex} = \frac{2T}{R}$

In soap bubble $P_{ex} = \frac{4T}{R}$

Angle of Contact (θ_C)

The angle enclosed between the tangent plane at the liquid surface and the tangent plane at the solid surface at the point of contact inside the liquid is defined as the angle of contact.

The angle of contact depends on the nature of the solid and liquid in contact.

Angle of contact $\theta < 90^\circ \Rightarrow$ concave shape, Liquid rise up in capillary

Angle of contact $\theta > 90^\circ \Rightarrow$ convex shape, Liquid falls down in capillary

Angle of contact $\theta = 90^\circ \Rightarrow$ plane shape, Liquid neither rise nor falls

Capillary rise $h = \frac{2T\cos\theta}{r\rho g}$

When two soap bubbles are in contact then radius of curvature of the common surface. $r = \frac{r_1 r_2}{r_1 - r_2} (r_1 > r_2)$

When two soap bubbles are combined to form a new bubble then radius of new bubble. $r = \sqrt{r_1^2 + r_2^2}$

VISCOSITY

Newton's law of viscosity $F = \eta A \frac{\Delta V_x}{\Delta y}$

SI UNITS of η : $\frac{N \times s}{m^2}$

CGS UNITS : dyne–s/cm^2 or poise (1 decapoise = 10 poise)

Dependency of viscosity of fluids On Temperature of Fluid

(a) Since cohesive forces decrease with increase in temperature as increase in K.E.. Therefore with the rise in temperature, the viscosity of liquids decreases.

(b) The viscosity of gases is the result of diffusion of gas molecules from one moving layer to other moving layer. Now with increase in temperature, the rate of diffusion increasers. So, the viscosity also increases. Thus, the viscosity of gases increase with the rise of temperature.

On Pressure of Fluid

The viscosity of liquid increases with the increase of pressure and the viscosity of gases is practically independent of pressure.

On Nature of Fluid

Poiseuille's formula $Q = \frac{dV}{dt} = \frac{\pi p r^4}{8\eta L}$

Viscous force $F_V = 6\pi\eta r v$

Terminal velocity $V_T = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta} \Rightarrow V_T \alpha r^2$

Reynolds number $R_e = \frac{\rho V d}{\eta}$

$R_e < 1000$ laminar flow, $R_e > 2000$ turbulent flow

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CHAPTER 12

Thermal Physics

Part A
KINETIC THEORY

- **Boyle's law:** If *m* and *T* are constant

$$V \propto \frac{1}{P}$$

$$P_1V_1 = P_2V_2$$

- **Charles's law:** If *m* and *P* are constant

$$V \propto T$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

- **Gay-Lussac's law:** If *m* and *V* are constant

$$P \propto T$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

- **Avogadro's law:** If *P*, *V* and *T* are same

$$N_1 = N_2$$

where, N_1 and N_2 are the number of molecules

$$V \propto n \text{ (no. of molecules of gas)}$$

$$\frac{V_1}{n_1} = \frac{V_2}{n_2}$$

- **Graham's law:** If *P* and *T* are constant

$$\text{rate of diffusion } r \propto \frac{1}{\sqrt{\rho}}$$

ρ is density

$$\frac{r_1}{r_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

- **Dalton's law:** $P = P_1 + P_2 + P_3$ ......

  P = Total pressure

  P_1, P_2, P_3 .... = Pressure exerted by each component present in the mixture.

- **Ideal gas equation:** $PV = \mathrm{n}RT = k_B NT$

  n = number of moles

  N = number of molecules

  R = universal gas constant

  k_B = Boltzmann's constant

- Pressure exerted by ideal gas

$$P = \frac{1}{3}\frac{mN}{v}\overline{v^2}$$

$\overline{v^2}$ = mean square velocity $\left\{\overline{v^2} = \frac{v_1^2 + v_2^2 + v_3^2 + ...}{N}\right\}$

m = mass of each molecule

or $P = \frac{1}{3}nm\overline{v^2}$

n = number density *i.e.*, $n = \frac{N}{V}$

- $$V_{ms} = \left(\overline{v^2}\right)^{1/2}$$

$$\text{K.E.} = \frac{3}{2}k_B T = \frac{1}{2}m\overline{v^2}$$

$$\overline{v^2} = \frac{3k_B T}{m}$$

$$\Rightarrow \mathrm{V}_{rms} = \sqrt{\frac{3k_B T}{m}}$$

- $$\mathrm{V}_{av} = \sqrt{\frac{8K_B T}{\pi m}}$$

- $$\mathrm{V}_{mp} = \sqrt{\frac{2K_B T}{m}}$$

- Mean free path $(\bar{l}) = \dfrac{1}{\sqrt{2}n\pi d^2}$

  n = number density

  d = diameter of molecule

- $\gamma = \dfrac{C_p}{C_v}$

  γ = ratio of specific heats

  C_p = specific heat at constant pressure

  C_v = specific heat at constant volume

  $C_p - C_v = R$

  R = universal gas constant

S.No.	Atomicity	No. of degree of freedom	C_P	C_v	$\gamma = C_P/C_v$
1	Monoatomic	3	$\frac{5}{2}R$	$\frac{3}{2}R$	$\frac{5}{3}$
2	Diatomic	5	$\frac{7}{2}R$	$\frac{5}{2}R$	$\frac{7}{5}$

- $C_{v(mix)} = \dfrac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2}$

  where n_1 and n_2 are number of moles of gases mixed together C_{v_1} and C_{v_2} are molar specific heat at constant volume of the two gas and $C_{v(mix)}$: Molar specific heat at constant volume for mixture.

Part B
THERMODYNAMICS

- First law of themodynamics

  $\Delta Q = \Delta U + \Delta W$

- Work done.

  $\Delta W = P\Delta V$

  $\therefore \Delta Q = \Delta U + P\Delta V$

- Relation between specific heats for a gas

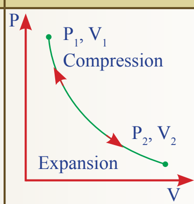
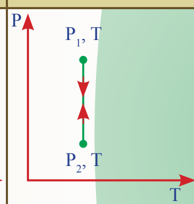
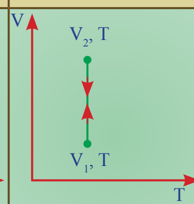
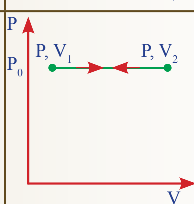
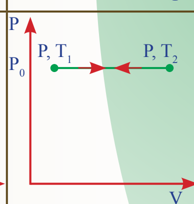
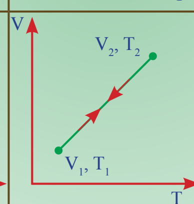
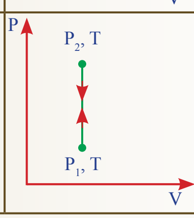
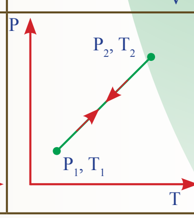
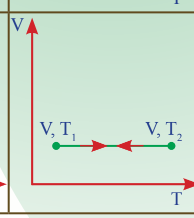
$$C_p - C_v = R$$

Polytropic Process

It is a thermodynamic process that can be expressed as follows:

$$PV^x = \text{Constant}$$

x (Polytropic exponent)	Type of standard process	Expression
0	Isobaric (dP = 0)	P = Constant
1	Isothermal (dT = 0)	PV = Constant
γ	Adiabatic (dQ = 0)	PV^γ = Constant
∞	Isochoric (dV = 0)	V = Constant

Process	Definition	P-V graph	P-T graph	V-T graph
Isothermal	Temperature constant			
Isobaric	Pressure constant			
Isochoric	Volume constant			

For any General Process

$$\Delta Q = \Delta U + W$$

$$\Rightarrow \qquad nC\Delta T = \frac{f}{2}nR\Delta T + \frac{nR\Delta T}{1-x}$$

[$\because$ Work done in a general polytropic process = [nRΔT/(1 – *x*)]

$$\Rightarrow \qquad C = \frac{f}{2}R + \frac{R}{1-x}$$

For infinitesimal changes in Q, U, and W, we ca write,

$$dQ = dU + dW$$

$$\Rightarrow \qquad nCdT = \frac{f}{2}nRdT + PdV$$

$$\Rightarrow \qquad C = \frac{f}{2}R + \frac{P}{n}\frac{dV}{dT}$$

Process	**Equation of State**	**W**	**ΔU**
Isobaric (dP = 0)	$\frac{V}{T} = c$	$P(V_f - V_i) = nR(T_f - T_i)$	$\frac{f}{2}nR(T_f - T_i) = \frac{f}{2}P(V_f - V_i)$
Isochoric (dV = 0)	$\frac{P}{T} = c$	0	$\frac{f}{2}nR(T_f - T_i) = \frac{f}{2}V(P_f - P_i)$
Isothermal (dT = 0)	$PV = c$	$nRT\ln\left(\frac{V_f}{V_i}\right) = nRT\ln\left(\frac{P_i}{P_f}\right)$	0
Adiabatic (dQ = 0)	$PV^{\gamma} = c$	$\frac{f}{2}nR(T_i - T_f)$ $= \frac{f}{2}(P_iV_i - P_fV_f)$	$\frac{f}{2}nR(T_f - T_i)$

Process	**ΔQ**
Isobaric (dP = 0)	$\left(\frac{f}{2}+1\right)nR\Delta T = \left(\frac{f}{2}+1\right)P(V_f - V_i)$
Isochoric (dV = 0)	$\frac{f}{2}nR(T_f - T_i) = \frac{f}{2}V(P_f - P_i)$
Isothermal (dT = 0)	$nRT\ln\left(\frac{V_f}{V_i}\right) = nRT\ln\left(\frac{P_i}{P_f}\right)$
Adiabatic (dQ = 0)	0

- Slope of adiabatic = γ (slope of isotherm)
- Carnot engine

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$W = Q_1 - Q_2$$

$$\text{(efficiency) } \eta = \frac{W}{Q_1}$$

- Refrigerator

Coefficient of performance is β

$$\beta = \frac{Q_2}{Q_1 - Q_2} = \frac{Q_2}{W}$$

$$\beta = \frac{1-\eta}{\eta}$$

- Heat pump

$$\alpha = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} = \frac{1}{\eta}$$

SUMMARY

- Zeroth law of thermodynamics states that 'two systems in thermal equilibrium with a third system are in thermal equilibrium with each other.
- Zeroth law leads to the idea of temperature.
- Heat and work are two modes of energy transfer to the system.
- Heat gets transferred due to temperature difference between the system and its environment (surroundings).
- Work is energy transfer which arises by other means, such as moving the piston of a cylinder containing the gas, by raising or lowering some weight connected to it.
- Internal energy of any thermodynamic system depends only on its state. The internal energy change in a process depends only on the initial and final states, not on the path, i.e. it is state dependent.

- The internal energy of an isolated system is constant.
- (*a*) For isothermal process $\Delta T = 0$

  (*b*) For adiabatic process $\Delta Q = 0$
- First law of thermodynamics states that when heat Q is added to a system while the system does work W, the internal energy U changes by an amount equal to $Q - W$. This law can also be expressed for an infinitesimal process.
- First law of thermodynamics is general law of conservation of energy.
- Second law of thermodynamics does not allow some processes which are consistent with the first law of thermodynamics.

  It states

  Clausius statement: No process is possible whose only result is the transfer of heat from a colder object to a hotter object.

  Kelvin-Plank statement: No process is possible whose only result is the absorption of heat from a reservoir and complete conversion of the heat into work.
- No engine can have efficiency equal to 1 or no refrigerator can have co-efficient of performance equal to infinity.
- Carnot engine is an ideal engine.
- The Carnot cycle consists of two reversible isothermal process and two reversible adiabatic process.
- If $Q > 0$, heat is added to the system.

  If $Q < 0$, heat is removed from the system.

  If $W > 0$, work is done by the system (Expansion).

  If $W < 0$, work is done on the system (Compression).

Part C

THERMAL PROPERTIES OF MATTER

- (*a*) $T_C = \frac{5}{9}(T_F - 32)$

  (*b*) $T = T_C + 273.15$

  (*c*) $T_F = \frac{9}{5}T - 459.67$

  where T, T_C, T_F, stand for temperature readings on Kelvin scale, Celsius scale, Fahrenheit scale respectively.
- (*a*) $\alpha = \frac{\Delta L}{L\Delta T}$

(*b*) $\beta = \dfrac{\Delta A}{A\Delta T}$

(*c*) $\gamma = \dfrac{\Delta V}{V\Delta T}$

where ΔL, ΔA, ΔV represent the change in length, change in area and change in volume respectively, due to a change in temperature ΔT. Here L, A and V stand for original length, original area, original volume respectively. α, β and γ denote coefficients of linear, area and volume expansions.

- $\alpha = \beta/2 = \gamma/3$
- Thermal stress $\left(\dfrac{F}{A}\right) = Y\alpha\Delta T$
- $Q = mc\,\Delta T$

  where Q is the heat required to raise the temperature by ΔT of a substance of mass m and of specific heat c.
- $Q = \mathrm{mL}$

  where Q is the amount of heat required for changing the phase of pure substance of mass m and L is the latent heat of the substance.

  The amount of heat required to change the phases (state) of a unit mass of a substance without any change in its temperature and pressure is called its latent heat. This is referred as the latent heat of fusion (L_F) when the phase change is from solid to liquid ; and latent heat of vaporisation (L_v) when the phases change is from liquid to gas.
- (*a*) $Q = \dfrac{kA(T_1 - T_2)t}{x}$

  $$H = kA\left(\frac{T_1 - T_2}{L}\right)$$

  where Q is the amount of heat that flows in time t across the opposite faces of a rod of length x and cross-section A. T_1 and T_2 are the temperatures of the faces in the steady state and k is the coefficient of thermal conductivity of the material of the rod.

  (*b*) $Q = -\,kA\left(\dfrac{dT}{dx}\right)t$

  where ($dTldx$) represents the temperature gradient.

  (*c*) $\mathrm{H} = \dfrac{dQ}{dt} = -\,\mathrm{kA}\left(\dfrac{dT}{dx}\right)$

  H is called the heat current.

- "Newton's Law of Cooling" says that the rate of cooling of a body is proportional to the excess temperature of the body over its surroundings:

$$\frac{dQ}{dt} = -k(T_2 - T_1)$$

where T_1 is the temperature of the surrounding medium and T_2 is the temperature of the body.

- (*a*) $r = \frac{Q_1}{Q}$

(*b*) $a = \frac{Q_2}{Q}$

(*c*) $t = \frac{Q_3}{Q}$

where Q_1 is the radiant energy reflected, Q_2 is the radiant energy absorbed and Q_3 is the radiant energy transmitted through a surface on which Q is the incident radiant energy. Further *r*, *a* and *t* denote reflectance, absorptance and transmittance of the surface.

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Chapter 13

Simple Harmonic Motion

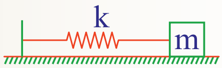
SIMPLE HARMONIC MOTION

$$F = -kx$$

General equation of S.H.M. is $x = A \sin(\omega t + \phi)$; $(\omega t + \phi)$ is phase of the motion and ϕ is initial phase of the motion.

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Time period (T) : $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$



Speed : $v = \omega\sqrt{A^2 - x^2}$

Acceleration : $a = -\omega^2 x$

Kinetic Energy (KE) : $\frac{1}{2}mv^2 = \frac{1}{2}m\omega^2\left(A^2 - x^2\right) = \frac{1}{2}k\left(A^2 - x^2\right)$

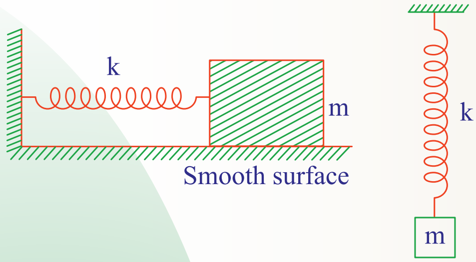
Potential Energy (PE) : $\frac{1}{2}Kx^2$

Total Mechanical Energy (TME)

$$= \text{K.E.} + \text{P.E.} = \frac{1}{2}k(A^2 - x^2) + \frac{1}{2}Kx^2 = \frac{1}{2}KA^2$$

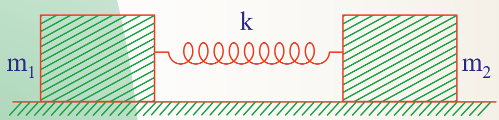
(which is constant)

SPRING-MASS SYSTEM

1. 

$$T = 2\pi\sqrt{\frac{m}{k}}$$

2. $T = 2\pi\sqrt{\frac{m}{k}}$,where $\mu = \frac{m_1 m_2}{(m_1 + m_2)}$ is known as "reduced mass".



COMBINATION OF SPRINGS

Series Combination : $1/K_S = 1/K_1 + 1/K_2$

Parallel combination : $K_P = K_1 + K_2$

Simple pendulum $T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{\ell}{g_{eff.}}}$ (in accelerating Reference Frame); g_{eff} is net acceleration due to psuedo force and gravitational force.

COMPOUND PENDULUM/PHYSICAL PENDULUM

TIME PERIOD (*T*):

$$T = 2\pi\sqrt{\frac{I}{mg\ell}}$$

where, $I = I_{cm} + m\ell^2$; ℓ is distance between point of suspension and centre of mass.

TORSIONAL PENDULUM

Time period (T) : $T = 2\pi\sqrt{\frac{I}{C}}$ where, C = Torsional constant

Superposition of two SHM s along the same direction

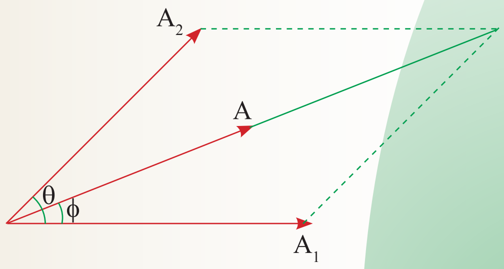
$$x_1 = A_1 \sin \omega t$$

and $$x_2 = A_2 \sin (\omega t + \theta)$$

If equation of resultant SHM is taken as $x = A \sin (\omega t + \phi)$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\theta}$$

and $$\tan\phi = \frac{A_2 \sin\phi}{A_1 + A_2 \cos\phi}$$



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CHAPTER 14

String Waves

GENERAL EQUATION OF WAVE MOTION

$$\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}$$

$$y(x,t) = f\left(t \pm \frac{x}{v}\right)$$

where, y(x, t) should be finite everywhere.

$f\left(t + \frac{x}{v}\right)$ represents wave travelling in –ve x-axis.

$f\left(t - \frac{x}{v}\right)$ represents wave travelling in + ve x-axis.

$y = A \sin(\omega t \pm kx + \phi)$

TERMS RELATED TO WAVE MOTION (For 1-D Progressive Sine Wave)

Wave Number (or Propagation Constant) (k)

$$k = 2\pi/\lambda = \frac{\omega}{v}\left(\text{rad m}^{-1}\right)$$

Phase of Wave

The argument of harmonic function ($\omega t \pm kx + \phi$) is called phase of the wave.

Phase difference ($\Delta\phi$) : difference in phases of two particles at any time t.

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x \quad \text{where } \Delta x \text{ is path difference}$$

Also $$\Delta\phi = \frac{2\pi}{T}.\Delta t$$

Speed of Transverse Wave Along the String

$$v = \sqrt{\frac{T}{\mu}}$$ where T = Tension

μ = mass per unit length

Power Transmitted Along the String

Average Power $$\langle P \rangle = 2\pi^2 f^2 A^2 \mu v$$

Intensity $$I = \frac{\langle P \rangle}{s} = 2\pi^2 f^2 A^2 \rho v$$

REFLECTION OF WAVES

If we have a wave

$$y_i(x, t) = a \sin(kx - wt) \text{ then,}$$

(i) Equation of wave reflected at a rigid boundary

$$y_r(x, t) = a \sin(kx + wt + \pi)$$

or $$y_r(x, t) = -a \sin(kx + wt)$$

i.e. the reflected wave is 180° out of phase.

(ii) Equation of wave reflected at an open boundary

$$y_r(x, t) = a \sin(kx + wt)$$

i.e. the reflected wave is in phase with the incident wave.

STANDING/STATIONARY WAVES

$$y_1 = A \sin(\omega t - kx + \theta_i)$$

$$y_2 = A \sin(\omega t - kx + \theta_2)$$

$$y_1 + y_2 = 2A\cos\left(kx + \frac{\theta_2 - \theta_1}{2}\right)\sin\left(\omega t + \frac{\theta_1 + \theta_2}{2}\right)$$

The quantity $2A\cos\left(kx + \frac{\theta_2 - \theta_1}{2}\right)$ represents resultant amplitude at x. At some position resultant amplitude is zero these are called **nodes**. At some positions resultant amplitude is 2A, these are called **antinodes**.

Distance between successive nodes or antinodes $= \frac{\lambda}{2}$

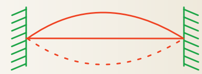
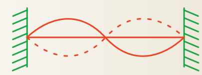
Distance between adjacent nodes and antinodes = $\lambda/4$.

All the particles in same segment (portion between two successive nodes) vibrate in same phase.

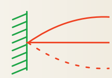
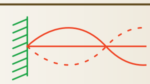
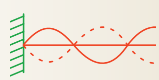
Since nodes are permanently at rest so energy can not be transmitted across these.

VIBRATIONS OF STRINGS (STANDING WAVE)

Fixed at Both Ends

First harmonics or Fundamental frequency	$L = \frac{\lambda}{2},\ f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$	
Second harmonics or First overtone	$L = \frac{2\lambda}{2},\ f_2 = \frac{2}{2L}\sqrt{\frac{T}{\mu}}$	
Third harmonics or Second overtone	$L = \frac{3\lambda}{2},\ f_3 = \frac{3}{2L}\sqrt{\frac{T}{\mu}}$	
n^{th} harmonics or $(n-1)^{th}$ overtone	$L = \frac{n\lambda}{2},\ f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$	

String Free at One End

First harmonics or Fundamental frequency	$L = \frac{\lambda}{4},\ f_1 = \frac{1}{4L}\sqrt{\frac{T}{\mu}}$	
Third harmonics or First overtone	$L = \frac{3\lambda}{4},\ f_3 = \frac{3}{4L}\sqrt{\frac{T}{\mu}}$	
Fifth harmonics or Second overtone	$L = \frac{5\lambda}{4},\ f_5 = \frac{5}{4L}\sqrt{\frac{T}{\mu}}$	
$(2n+1)^{th}$ harmonic or n^{th} overtone	$L = \frac{(2n+1)\lambda}{4},$ $f_{2n+1} = \frac{(2n+1)}{4L}\sqrt{\frac{T}{\mu}}$	

CHAPTER 15

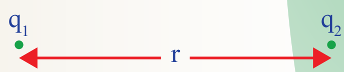
Electrostatics

ELECTRIC CHARGE

Charge of a meterial body is that property due to which it interacts with other charges. There are two kinds of charges- positive and negative. SI unit is coulomb. Charge is quantized and additive.

Coulomb's law:

Force between two charges $\vec{F} = \frac{1}{4\pi \in_0 \in_r} \frac{q_1 q_2}{r^2} \hat{r}$ $\in_r$ = dielectric constant



NOTES

The Law is applicable only for static point charges.

Principle of Superposition

Force on a point charge due to many charges is given by

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots..$$

NOTES

The force due to one charge is not affected by the presence of other charges.

Electric Field or Electric Field Intensity (Vector Quantity)

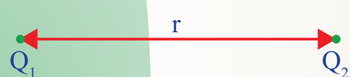
In the surrounding region of a charge there exist a physical property due to which other charges experience a force. The direction of electric field is direction of force experienced by a positively charged particle and magnitude of the field (electric field intenity) is the force experienced by a unit charge.

$$\vec{E} = \frac{\vec{F}}{q} \text{ unit is N/C or V/m.}$$

Electric field due to charge Q

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \frac{1}{4\pi \in_0} \frac{Q}{r^2} \hat{r}$$

Null point for two charges :



If $|Q_1| > |Q_2|$

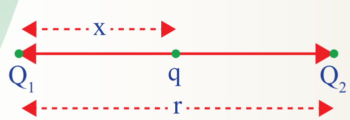
$\Rightarrow$ Null point near Q_2

$$x = \frac{\sqrt{Q_1}\, r}{\sqrt{Q_1} \pm \sqrt{Q_2}}; \; x \to \text{distance of null point from } Q_1 \text{ charge}$$

(+) for like charges

(–) for unlike charges

Equilibrium of three point charges



(i) Two charges must be of like nature.

(ii) Third charge should be of unlike nature.

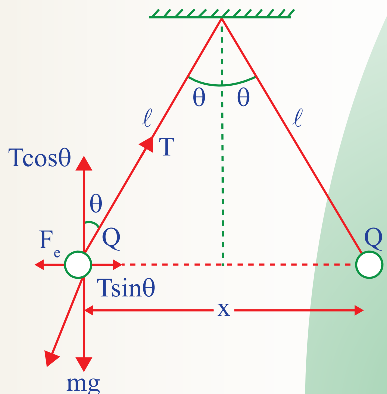
$$x = \frac{\sqrt{Q_1}}{\sqrt{Q_1} + \sqrt{Q_2}} r \quad \text{and} \quad q = \frac{-Q_1 Q_2}{\left(\sqrt{Q_1} + \sqrt{Q_2}\right)^2}$$

Equilibrium of suspended point charge system

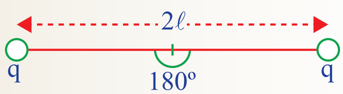
For equilibrium position

$$T\cos\theta = mg \;\&\; T\sin\theta = F_e = \frac{kQ^2}{x^2} \Rightarrow \tan\theta = \frac{F_e}{mg} = \frac{kQ^2}{x^2 mg}$$

$$T = \sqrt{(F_e)^2 + (mg)^2}$$



If whole set up is taken into an artificial satellite ($g_{eff} \simeq 0$)



$$T = F_e = \frac{kq^2}{4\ell^2}$$

Electric potential difference $\Delta V = \frac{\text{work}}{\text{charge}} = W/q$

Electric potential $V_p = -\int_{\infty}^{P} \vec{E}.d\vec{r}$

It is the work done against the field to take a unit positive charge from infinity (reference point) to the given point

- For point charge : $V = K\frac{q}{r}$
- For several point charges : $V = K\sum \frac{q_1}{r_1}$

Relation between $\vec{E}$ & V

$$\vec{E} = -\text{grad}\, V = -\nabla V, E = \frac{-\partial V}{\partial r}; \quad \vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}, V = \int -\vec{E}.d\vec{r}$$

Electric potential energy of two charges : $U = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r}$

Electric dipole

- Electric dipole moment p = qd
- Torque on dipole placed in uniform electric field $\vec{\tau} = \vec{p} \times \vec{E}$
- Work done in rotating dipole placed in uniform electric field

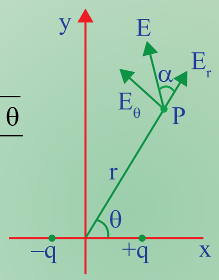
$$W = \int \tau d\theta = \int_{\theta_0}^{\theta} PE \sin\theta d\theta = pE(\cos\theta_0 - \cos\theta)$$

- Potential energy of dipole placed in an uniform field $U = -\vec{p}.\vec{E}$
- At a point which is at a distance r from dipole midpoint and making angle θ with dipole axis.

Potential $V = \frac{1}{4\pi \in_0} \frac{p\cos\theta}{r^2}$

Electric field $E = \frac{1}{4\pi \in_0} \frac{p\sqrt{1+3\cos^2\theta}}{r^3}$

Direction $\tan\alpha = \frac{E_\theta}{E_r} = \frac{1}{2}\tan\theta$



- Electric field at axial point (or End-on) $\vec{E} = \frac{1}{4\pi \in_0} \frac{2\vec{p}}{r^3}$ of dipole
- Electric field at equatorial position (Broad-on) of dipole $\vec{E} = \frac{1}{4\pi \in_0} \frac{(-\vec{p})}{r^3}$

Equipotential Surface and Equipotential Region

In an electric field the locus of points of equal potential is called an equipotential surface. An equipotential surface and the electric field line meet at right angles. The region where E = 0, Potential of the whole region must remain constant as no work is done in displacement of charge in it. It is called as equipotential region like conducting bodies.

Electric flux : $\phi = \int \vec{E}.\vec{ds}$

Gauss's Law : $\oint \vec{E}.\vec{ds} = \frac{\sum q}{\in_0}$ (Applicable only on closed surface)

Net flux emerging out of a closed surface is $\frac{q_{en}}{\varepsilon_0}$

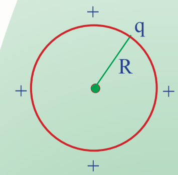
$\phi = \oint \vec{E}.d\vec{A} = \frac{q_{en}}{\varepsilon_0}$ where q_{en} = net charge enclosed by the closed surface.

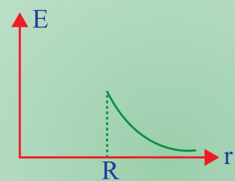
ϕ does not depend on the

(i) Shape and size of the closed surface

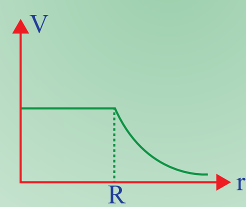
(ii) The charges located outside the closed surface.

For a conducting sphere



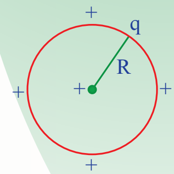


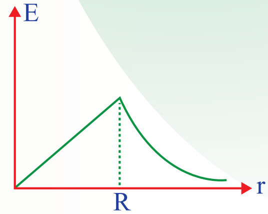
For $r \geq R : E = \frac{1}{4\pi \in_0}\frac{q}{r^2}$, $V = \frac{1}{4\pi \in_0}\frac{q}{r}$



For $r < R : E = 0,\ V = \frac{1}{4\pi \in_0}\frac{q}{R}$

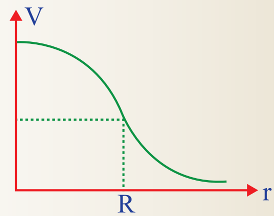
For a non-conducting sphere





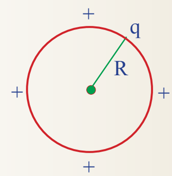
For $r \geq R : E = \frac{1}{4\pi \in_0}\frac{q}{r^2}, V = \frac{1}{4\pi \in_0}\frac{q}{r}$

For $r < R : E = \frac{1}{4\pi\epsilon_0}\frac{qr}{R^3}$, $V = \frac{1}{4\pi\epsilon_0}\frac{q(3R^2 - r^2)}{2R^3}$



$$V_c = V_{max} = \frac{3}{2}\frac{Kq}{R} = 1.5V_{surface}$$

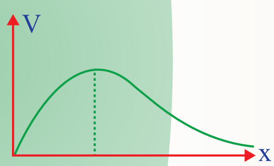
For a conducting/non conducting spherical shell

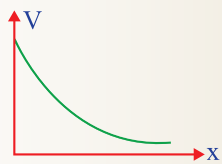


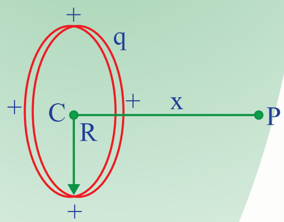
For $r \geq R : E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$, $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

For $r < R : E = 0$, $V = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$

For a charged circular ring



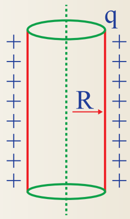




$E_P = \frac{1}{4\pi\epsilon_0}\frac{qx}{(x^2 + R^2)^{3/2}}$, $V_p = \frac{q}{4\pi\epsilon_0 (x^2 + R^2)^{1/2}}$

Electric field will be maximum at $x = \pm\frac{R}{\sqrt{2}}$

For a charged long conducting cylinder



- For $r \geq R : E = \frac{q}{2\pi\epsilon_0 r}$
- For $r < R : E = 0$

Electric field intensity at a point near a charged conductor $E = \dfrac{\sigma}{\epsilon_0}$

Mechanical pressure on a charged conductor $P = \dfrac{\sigma^2}{2\epsilon_0}$

For non-conducting infinite sheet of surface charged density $E = \dfrac{\sigma}{2\epsilon_0}$

For conducting infinite sheet of surface charge density $E = \dfrac{\sigma}{\epsilon_0}$

Energy density in electric field $U = \dfrac{\epsilon_0}{2}E^2$

Electric lines of force

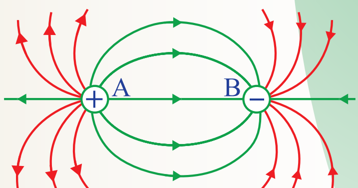
Electric lines of electrostatic field have following properties.

(i) Imaginary

(ii) Can never cross each other

(iii) Can never be closed loops

(iv) The number of lines originating or terminating on a charge is proportional to the magnitude of charge.



$q_A > q_B$

(v) Lines of force ends or starts normally at the surface of a conductor.

(vi) If there is no electric field there will be no lines of force.

(vii) Lines of force per unit area normal to the area at a point represents magnitude of intensity, crowded lines represent strong field while distant lines weak field.

(viii) Tangent to the line of force at a point in an electric field gives the direction of Electric Field.

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CHAPTER 16

Capacitance

CAPACITOR & CAPACITANCE

A capacitor consists of two conductors carrying charges of equal magnitude and opposite sign. The capacitance C of any capacitor is the ratio of the charge Q on either conductor to the potential difference V between them $C = \frac{Q}{V}$

The capacitance depends only on the geometry of the conductors.

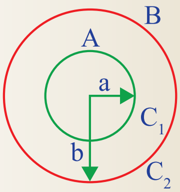
Capacitance of an Isolated Spherical Conductor

$C = 4\pi \in_0 \in_r R$ in a medium $C = 4\pi \in_0 R$ in air.

This sphere is at infinite distance from all the conductors.

Spherical Capacitor :

It consists of two concentric spherical shells as shown in figure. Here capacitance of region between the two shells is C_1 and that outside the shell is C_2. We have $C_1 = \frac{4\pi \in_0 ab}{b-a}$ and $C_2 = 4\pi\in_0 b$.

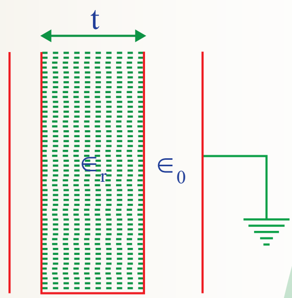


Parallel Plate Capacitor

(i) Uniform Di-Electric Medium : If two parallel plates each of area A & separated by a distance d are charged with equal & opposite charge Q, then the system is called a **parallel plate capacitor** & its capacitance is given by, $C = \frac{\in_0 \in_r A}{d}$ in a medium, $C = \frac{\in_0 A}{d}$ with air as medium.

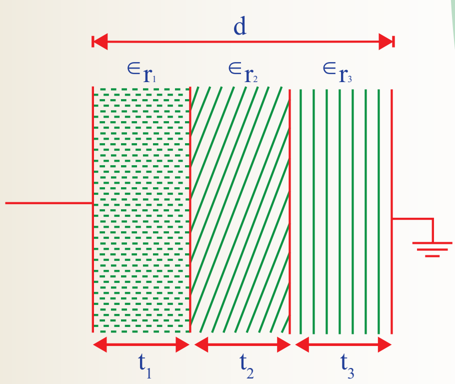
This result is only valid when the electric field between plates of capacitor is constant.

(ii) Medium Partly Air : $C = \dfrac{\epsilon_0 A}{d - \left(t - \dfrac{t}{\epsilon_r}\right)}$



When a di-electric slab of thickness t & relative permitivity ϵ_r is introduced between the plates of an air capacitor, then the distance between the plates is effectively reduced by $\left(t - \dfrac{t}{\epsilon_r}\right)$ irrespective of the position of the di-electric slab.

(iii) Composite Medium :

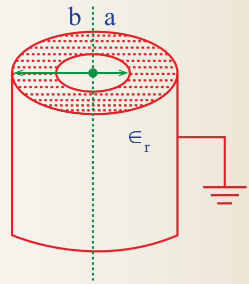


$$C = \frac{\epsilon_0 A}{\dfrac{t_1}{\epsilon_{r1}} + \dfrac{t_2}{\epsilon_{r2}} + \dfrac{t_3}{\epsilon_{r3}}}$$

Cylindrical Capacitor :

It consists of two co-axial cylinders of radii a & b, the outer conductor is earthed. The di-electric constant of the medium filled in the space between the cylinders is ϵ_r.

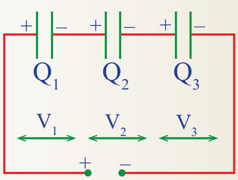
The capacitance per unit length is $C = \dfrac{2\pi \in_0 \in_r}{\ell n\left(\dfrac{b}{a}\right)}$



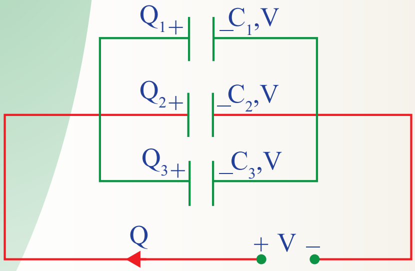
Combination of Capacitors :

(i) Capacitors in Series : In this arrangement all the capacitors when uncharged get the same charge Q but the potential difference across each will differ (if the capacitance are unequal).

$$\frac{1}{C_{eq.}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n}$$



(ii) Capacitors in Parallel : When one plate of each capacitor is connected to the positive terminal of the battery & the other plate of each capacitor is connected to the negative terminals of the battery, then the capacitors are said to be parallel connection. The capacitors have the same potential difference V, but the charge on each one is different (if the capacitors are unequal). $C_{eq.} = C_1 + C_2 + C_3 + ... + C_n$.



Energy Stored in a Charged Capacitor

Capacitance C, Charge Q & potential difference V; then energy stored is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

This energy is stored in the electrostatic field set up in the di-electric medium between the conducting plates of the capacitor.

Heat Produced in Switching in Capacitive Circuit

Due to charge flow always some amount of heat is produced when a switch is closed in a circuit which can be obtained by energy conservation as

$$\text{Heat} = \text{Work done by battery} - \text{Energy absorbed by capacitor}$$

Work done by battery to charge a capacitor

$$W = CV^2 = QV = Q^2/C$$

Sharing of Charges: When two charged conductors of capacitance C_1 & C_2 at potential V_1 & V_2 respectively are connected by a conducting wire, the charge flows from higher potential conductor to lower potential conductor, until the potential of the two condensers become equal. The common potential (V) after sharing of charges;

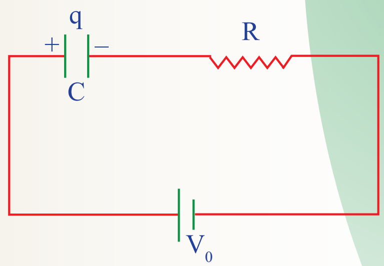
$$V = \frac{\text{net charge}}{\text{net capacitance}} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

Charges after sharing $q_1 = C_1V$ & $q_2 = C_2V$. In this process energy is lost in the connecting wire as heat.

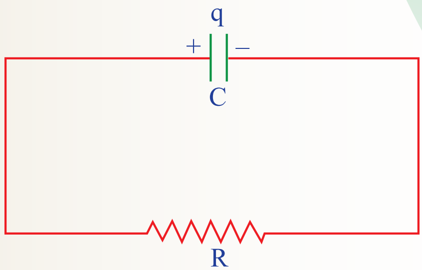
This loss of energy is $U_{\text{initial}} - U_{\text{final}} = \dfrac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$

Attractive force between capacitor plate: $F = \left(\dfrac{\sigma}{2\in_0}\right)(\sigma A) = \dfrac{Q^2}{2\in_0 A}$

Charging of a capacitor: $q = q_0\,(1 - e^{-t/RC})$ where $q_0 = CV_0$



Discharging of a capacitor : $q = q_0e^{-t/RC}$



KEY TIPS

- The energy of a charged conductor resides outside the conductor in its electric field, where as in a condenser it is stored within the condenser in its electric field.
- The energy of an uncharged condenser = 0.
- The capacitance of a capacitor depends only on its size & geometry & the dielectric between the conducting surface.
- The two adjacent conductors carrying same charge can be at different potential bacause the conductors may have different sizes and means different capacitance.
- On filling the space between the plates of a parallel plate air capacitor with a dielectric, capacity of the capacitor is increased because the same amount of charge can be stored at a reduced potential.
- The potential of a grounded object is taken to be zero because capacitance of the earth is very large.

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CHAPTER 17

Current Electricity and Heating Effects of Current

ELECTRIC CURRENT

Electric charges in motion constitute an electric current. Positive charge flows from higher to lower potential and negative charge flows from lower to higher. Metals such as gold, silver, copper, aluminium etc., are good conductors. The strength of the current i is the rate at which the electric charges are flowing. If a charge Q coulomb passes through a given cross section of the conductor in t second the current I through the conductor is given by

$$I = \frac{Q \text{ coulomb}}{t \text{ second}} = \text{ampere}$$

Ampere is the unit of current. If i is not constant then $i = \frac{dq}{dt}$, where dq is net charge transported at a section in time dt.

Electric Current in A Conductor

When a potential difference is applied across a conductor the charge carriers (electrons in case of metallic conductors) flow in a definite direction which constitutes a net current in it. They move with a constant drift velocity. The direction of current is along the flow of positive charge (or opposite to flow of negative charge). $i = nv_d eA$, where v_d = drift velocity.

Current Density

In a current carrying conductor we can define a vector which gives the direction as current per unit normal, cross sectional area & is known as current density.

Thus $\vec{J} = \frac{I}{S}\hat{n}$ or $i = \vec{J}.\vec{S}$ where $\hat{n}$ is the unit vector in the direction of the flow of current.

RELATION IN J, E AND V_D

In conductors drift velocity of electrons is proportional to the electric field inside the conductor as; $v_D = \mu E$

where μ is the mobility of electrons

current density is given as $J = \frac{I}{A} = ne\, v_d = ne(\mu E) = \sigma E$

where $\sigma = ne\mu$ is called conductivity of material and we can also write $\rho = \frac{1}{\sigma} \rightarrow$ resistivity of material.

Thus $\vec{E} = \rho\, \vec{J}$. It is called as differential form of Ohm's Law.

Sources of Potential Difference and Electromotive Force

Dry cells, secondary cells, generator and thermo couple are the devices used for producing potential difference in an electric circuit. The potential difference between the two terminals of a source when no energy is drawn from it, is called the **"Electromotive force"** or **"EMF"** of the source. The unit of potential difference is volt.

1 volt = 1 Ampere x 1 Ohm.

ELECTRICAL RESISTANCE

The property of a substance which opposes the flow of electric current though it, is termed as electrical resistance. Electrical resistance depends on the size, geometry, temperature and internal structure of the conductor.

Law of Resistance

The resistance R offered by a conductor depends on the following factors :

$R \propto \ell$ (length of the conductor); $R \propto \frac{1}{A}$ (cross section area of the conductor)

at a given temperature $R = \rho\frac{\ell}{A}$ Where ρ is the resistivity of the material of the conductor at the given temperature. It is also known as **specific resistance** of the material & it depends upon nature of conductor.

Dependence of Resistance on Temperature

The resistance of most conductors and all pure metals increases with temperature. If R_0 & R be the resistance of a conductor at 0°C and θ°C, then it is found that $R = R_0(1 + \alpha\theta)$.

Instead of resistance we use same property for resistivity as $\rho = \rho_0\,(1 + \alpha\theta)$.

Where α is called the temperature co-efficient of resistance. The unit of α is K^{-1} or $°C^{-1}$. Reciprocal of resistivity is called conductivity and reciprocal of resistance is called conductance (G). S.I. unit of G is mho.

The materials for which resistance decreases with temperature, the temperature coefficient of resistance is negative.

OHM'S LAW

It says that the current through the cross section of the conductor is proportional to the applied potential difference under the given physical condition. V = RI. Ohm's law is applicable to only metallic conductors.

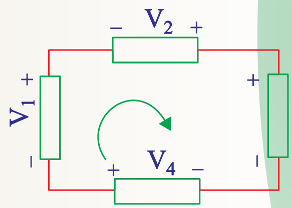
KRICHHOFF'S LAW'S

I - Law (Junction law or Nodal Analysis): This law is based on law of conservation of charge. It states that "The algebraic sum of the currents meeting at a point is zero" or total currents entering a junction equals total current leaving the junction.

$$\sum I_{in} = \sum I_{out}$$

It Is also known as KCL (Kirchhoff's current law).

II - Law (Loop analysis): The algebraic sum of all the voltages in closed circuit is zero. $\sum IR + \sum EMF = 0$ in a closed loop. The closed loop can be traversed in any direction. While traversing a loop if higher potential point is entered, put a positive sign in expression or if lower potential point is entered put a negative sign.



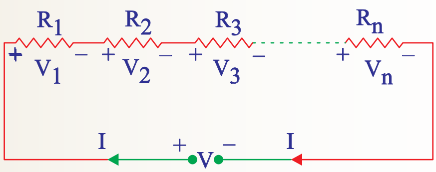
$-V_1 - V_2 + V_3 - V_4 = 0$. Boxes may contain resistor or battery or any other element (linear or non-linear).

It is also known as **KVL (Kirchhoff's voltage law).**

COMBINATION OF RESISTANCES

(i) Resistance In Series:

When the resistances are connected end to end then they are said to be in series. The current through each resistor is same. The effective resistance appearing across the battery;



$$R = R_1 + R_2 + R_3 + ...+ R_n \text{ and}$$

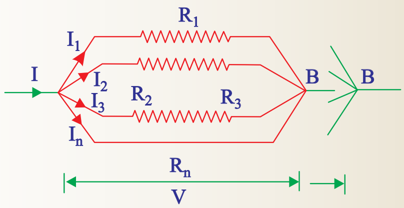
$$V = V_1 + V_2 + V_3 .... + V_n.$$

The voltage across a resistor is proportional to the resistance

$$V_1 = \frac{R_1}{R_1 + R_2 + ... + R_n} V; V_2 = \frac{R_2}{R_1 + R_2 + ... + R_n} V; \text{ etc.}$$

(ii) Resistance In Parallel:

A parallel circuit of resistors is one, in which the same voltage is applied across all the components.



Conclusions:

(a) Potential difference across each resistor is same.

(b) $I = I_1 + I_2 + I_3 + ... I_n$.

(c) Effective resistance (R) then $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... + \frac{1}{R_n}$.

(d) Current in different resistors is inversally proportional to the resistances.

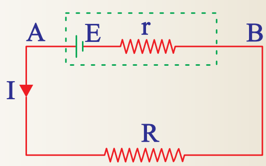
$$I_1 : I_2 : ....I_n = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} : .... : \frac{1}{R_n}$$

$$I_1 = \frac{G_1}{G_1 + G_2 + ... + G_n} I, I_2 = \frac{G_2}{G_1 + G_2 + ... + G_n} I, \text{ etc.}$$

where $G = \frac{I}{R} =$ Conductance of a resistor.

EMF OF A CELL & ITS INTERNAL RESISTANCE

If a cell of emf E and Internal resistance r be connected with a resistance R the total resistance of the circuit is (R + r).



$$I = \frac{E}{R + r}; V_{AB} = \frac{ER}{R + r}$$

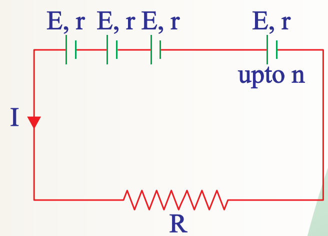
where V_{AB} = Terminal voltage of the battery.

If $r \to 0$, cell is ideal & $V \to E$ & $r = R\left(\frac{E}{V} - 1\right)$

GROUPING OF CELLS

(i) Cells In Series: Let there be n cells each of emf E, arranged in series. Let r be the internal resistance of each cell. The total emf = nE.

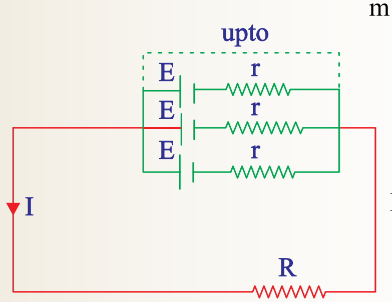
Current in the circuit $I = \frac{nE}{R + nr}$. If nr << R then $I = \frac{nE}{R}$.



If nr >> R then $I = \frac{E}{r}$.

(ii) Cells In Parallel: If m cells each of emf E & internal resistance r be connected in parallel and if this combination be connected to an external resistance then the emf of the circuit = E.

Internal resistance of the circuit = $\frac{r}{m}$.



$$I = \frac{E}{R + \frac{r}{m}} = \frac{mE}{mR + r}$$

If mR << r then $I = \frac{mE}{r}$.

If mR >> r then $I = \frac{E}{R}$.

(iii) Cells in Matrix Array:

n = number of rows

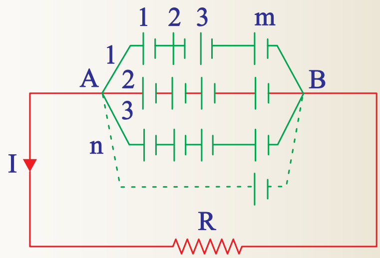
m = number of cells in each rows.

mn = number of identical cells.

The combination of cells is equivalent to single cell of:

(a) emf = mE &

(b) internal resistance $= \frac{mr}{n}$



Current $I = \frac{mE}{R + \frac{mr}{n}}$.

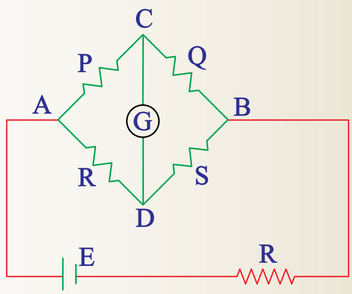
For maximum current

$nR = mr$ or $R = \frac{mr}{n}$

so $I_{max} = \frac{nE}{2r} = \frac{mE}{2R}$

For a cell to deliver maximum power across the load internal resistance = load resistance

WHEAT STONE NETWORK



When current through the galvanometer is zero (null point or balance point) $\frac{P}{Q} = \frac{R}{S}$.

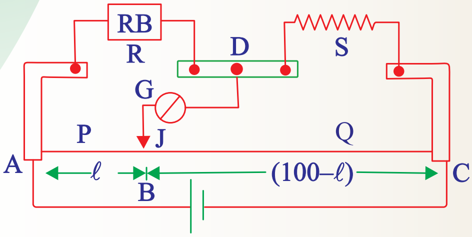
When,

$PS > QR$, $V_C < V_D$ & $PS < QR$, $V_C > V_D$ or $PS = QR \Rightarrow$ products of opposite arms are equal. Potential difference between C & D at null point is zero. The null point is not affected by resistance of G & E. It is not affected even if the positions of G & E are Interchanged.

$$I_{CD} \propto (QR - PS)$$

Metre Bridge

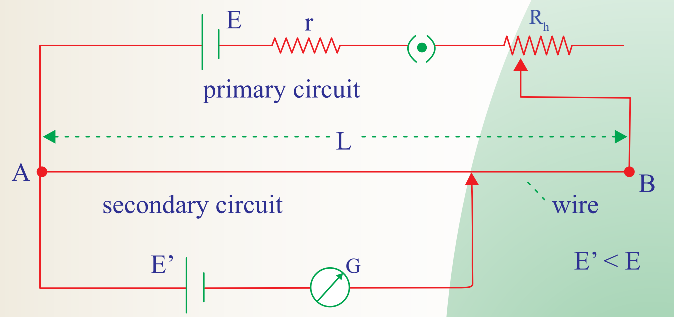
At balance condition : $\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{\ell}{(100-\ell)} = \frac{R}{S} \Rightarrow S = \frac{(100-\ell)}{\ell}R$



POTENTIOMETER

A potentiometer is a linear conductor of uniform cross-section with a steady current set up in it. This maintains a uniform potential gradient along the length of the wire. Any potential difference which is less than the potential difference maintained across the potentiometer wire can be measured using this. The potentiometer equation is $\frac{E_1}{E_2} = \frac{\ell_1}{\ell_2}$.

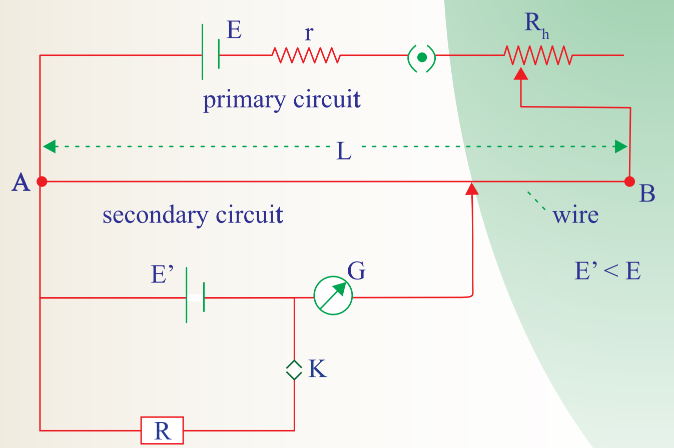
Circuits of potentiometer



Potential Gradient

$$\lambda = \frac{V_{AB}}{L} = \frac{\text{current} \times \text{resistance of potentiometer wire}}{\text{length of potentiometer wire}} = I\left(\frac{R}{L}\right)$$

Here the internal resistance of the cell is given by



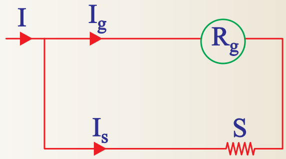
$$r = \left(\frac{\ell_1 - \ell_2}{\ell_2}\right) \cdot R$$

Where ℓ_1 and ℓ_2 are balancing lengths without shunt or with the shunt. R is the shunt resistance in parallel with the given cell.

AMMETER

It is used to measure current. A shunt (small resistance) is connected in parallel with galvanometer to convert into ammeter.

$$S = \frac{I_g R_g}{I - I_g}$$



where

R_g = galvanometer resistance

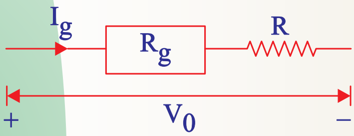
I_g = Maximum current that can flow through the galvanometer.

I = Maximum current that can be measured using the given ammeter.

An Ideal ammeter has zero resistance.

VOLTMETER

A high resistance is put in series with galvanometer. It is used to measure potential difference.



$$I_g = \frac{V_o}{R_g + R}; R \to \infty \text{, Ideal voltmeter}$$

$$R = (V/I_g) - R_g$$

ELECTRICAL POWER

The energy liberated per second in a device is called its power. The electrical power P delivered by an electrical device is given by P = VI, where V = potential difference across device & I = current. If the current enters the higher potential point of the device then power is consumed by It (i.e. acts as load). If the current enters the lower potential point then the device supplies power (i.e. acts as source).

Power consumed by a resistor

$$P = I^2R = VI = \frac{V^2}{R}$$

HEATING EFFECT OF ELECTRIC CURRENT

When a current is passed through a resistor, energy is wasted in over coming the resistances of the wire. This energy is converted into heat

$$W = VIt = I^2Rt = \frac{V^2}{R}t$$

JOULES LAW OF ELECTRICAL HEATING

The heat generated (in joules) when a current of I ampere flows through a resistance of R ohm for T second is given by:

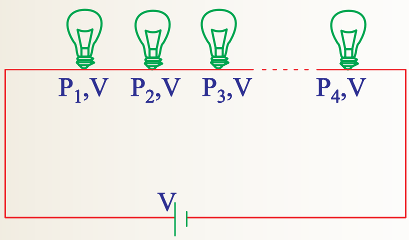
$$H = I^2 RT \text{ joule} = \frac{I^2RT}{4.2} \text{ calories}$$

If variable current passes through the conductor, then we use for heat produced in resistance from time 0 to T is; $H = \int_0^T I^2R\,dt$.

Unit of Electrical Energy Consumption

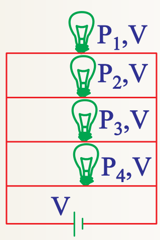
1 unit of electrical energy = kilowatt hour = 1 kWh = 3.6 x 10^6 joules.

- **Series combination of Bulbs**



$$\frac{1}{P_{total}} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} + ...$$

- **Parallel combination of Bulbs**



$$P_{total} = P_1 + P_2 + P_3 + ...$$

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CHAPTER 18

Magnetic Effect of Current and Magnetism

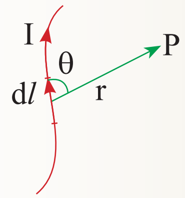
A static charge produces only electric field. A moving charge produces both electric field and magnetic field. A current carrying conductor produces only magnetic field.

MAGNETIC FIELD PRODUCED BY A CURRENT (BIOT-SAVART LAW)

The magnetic induction dB produced by an element dl carrying a current I at a distance r is given by:

$$dB = \frac{\mu_0\mu_r}{4\pi}\frac{I\,dl\sin\theta}{r^2} \Rightarrow d\vec{B} = \frac{\mu_o\mu_r}{4\pi}\frac{I\left(d\vec{l}\times\vec{r}\right)}{r^3}$$

here the quantity Idl is called as current element.



μ = permeability of the medium = $\mu_0\mu_r$

μ_0 = permeability of free space

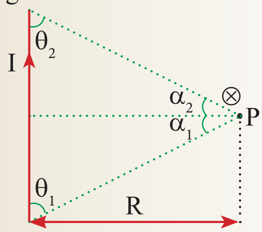
μ_r = relative permeability of the medium (Dimensionless quantity)

Unit of μ_0 & μ is NA^{-2} or Hm^{-1};

$\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$

Magnetic Induction Due To a Straight Current Conductor

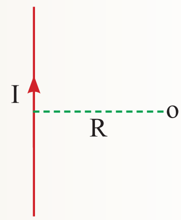
Magnetic induction due to a current carrying straight wire



$$B = \frac{\mu_0 I}{4\pi R}(\cos\theta_1 + \cos\theta_2) = \frac{\mu_0 I}{4\pi R}(\sin\alpha_1 + \sin\alpha_2)$$

Magnetic induction due to a infinitely long wire $B = \frac{\mu_0 I}{2\pi R} \otimes$

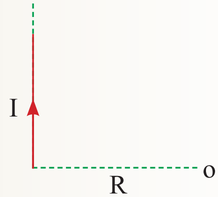
$\alpha_1 = 90^\circ$; $\alpha_2 = 90^\circ$



Magnetic induction due to semi infinite straight conductor

$$B = \frac{\mu_0 I}{4\pi R} \otimes$$

$\alpha_1 = 0^\circ$; $\alpha_2 = 90^\circ$



- Magnetic field due to a flat circular coil carrying a current:

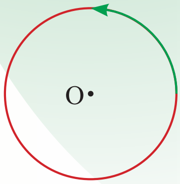
  (*i*) At its centre $B = \frac{\mu_0 NI}{2R} \odot$

  where

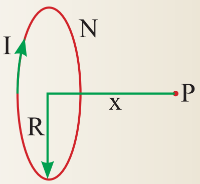
  N = total number of turns in the coil

  I = current in the coil

  R = Radius of the coil



(*ii*) On the axis $B = \dfrac{\mu_0 N I R^2}{2(x^2+R^2)^{3/2}}$

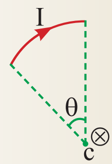


Where x = distance of the point from the centre.

It is maximum at the centre $B_C = \dfrac{\mu_0 N I}{2R}$

(*iii*) Magnetic field due to flat circular ARC :

$B = \dfrac{\mu_0 I \theta}{4\pi R}$



- Magnetic field due to infinite long solid cylindrical conductor of radius R

  - For $r \geq R : B = \dfrac{\mu_0 I}{2\pi r}$
  - For $r < R : B = \dfrac{\mu_0 I r}{2\pi R^2}$

Magnetic Induction Due to Solenoid

$B = \mu_0 n I$, direction along axis.

where $n \rightarrow$ number of turns per meter;

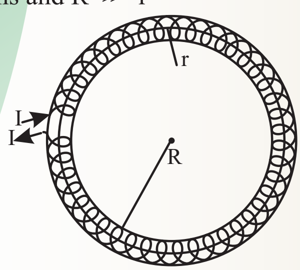
$I \rightarrow$ current

Magnetic Induction Due To Toroid :

$B = \mu_0 n I$

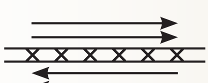
where $n = \dfrac{N}{2\pi R}$ (no. of turns per m)

N = total turns and $R \gg r$



Magnetic Induction Due To Current Carrying Sheet

B $\frac{1}{2}\mu_0\lambda$ where λ = Linear current density (A/m)



Earth's Magnetic Field

(a) Earth's magnetic axis is slightly inclined to the geometric axis of earth and this angle varies from 10.5º to 20º. The Earth's Magnetic poles are opposite to the geometric poles i.e. at earth's north pole, its magnetic south pole is situated and vice versa.

(b) On the magnetic meridian plane, the magnetic induction vector of the earth at any point, generally inclined to the horizontal at an angle called the **Magnetic Dip** at that place, such that $\vec{B}$ = total magnetic induction of the earth at that point.

$\vec{B}_v$ = the vertical component of in $\vec{B}$ in the magnetic meridian plane $= B \sin \theta$

$\vec{B}_H$ = the horizontal component of $\vec{B}$ in the magnetic meridian plane $= B \cos \theta$.

and $\tan \theta = \dfrac{B_v}{B_H}$

(c) At a given place on the surface of the earth, the magnetic merdian and the geographic meridian may not coincide. The angle between them is called "Declination" at that place.

AMPERES CIRCUITAL LAW

$\oint \vec{B}.\vec{d}l = \mu \Sigma I$ where Σ I = algebraic sum of all the current.

MOTION OF A CHARGE IN UNIFORM MAGNETIC FIELD:

(a) When $\vec{V}$ is || to $\vec{B}$; Motion will be in a straight line and $\vec{F} = 0$

(b) When $\vec{V}$ is $\perp$ to $\vec{B}$: Motion will be in circular path with radius $R = \dfrac{mv}{qB}$ and angular velocity $\omega = \dfrac{qB}{m}$ and $F = qvB$.

(c) When $\vec{V}$ is at $\angle \theta$ to $\vec{B}$: Motion will be helical with radius $R_k = \dfrac{mv\sin\theta}{qB}$ and pitch $P_H = \dfrac{2\pi mv\cos\theta}{qB}$ and $F = qvB\sin\theta$.

LORENTZ FORCE

An electric charge 'q' moving with a velocity $\vec{V}$ through a magnetic field of magnetic induction $\vec{B}$ experiences a force $\vec{F}$, given by $\vec{F} = q\vec{v} \times \vec{B}$. There fore, if the charge moves in a space where both electric and magnetic fields are superposed.

$\vec{F}$ = net electromagnetic force on the charge = $q\vec{E} + q\,\vec{v}\times\vec{B}$

This force is called the Lorentz Force

MOTION OF CHARGE IN COMBINED ELECTRIC FIELD & MAGNETIC FIELD

- When $\vec{v}\parallel\vec{B}$ & $\vec{v}\parallel\vec{E}$, Motion will be uniformly accelerated in line as $F_{magnetic} = 0$ and $F_{electrostatic} = qE$

  So the particle will be either speeding up or speeding down
- When $\vec{v}\parallel\vec{B}$ & $\vec{v}\perp\vec{E}$, motion will be uniformly accelerated in a parabolic path
- When $\vec{v}\perp\vec{B}$ & $\vec{v}\perp\vec{E}$, the particle will move undeflected & undeivated with same uniform speed if $v = \dfrac{E}{B}$ (This is called as velocity selector condition)

MAGNETIC FORCE ON A STRAIGHT CURRENT CARRYING WIRE : $\vec{F} = I\,(\vec{L}\times\vec{B})$

I = current in the straight conductor

$\vec{L}$ = length of the conductor in the direction of the current in it

$\vec{B}$ = magnetic induction. (Uniform throughout the length of conductor)

Note : In general force is $\vec{F} = \int I\left(d\vec{l}\times\vec{B}\right)$

Magnetic Interaction Force Between Two Parallel Long Straight Currents:

The interactive force between 2 parallel long straight wires is:

(i) Repulsive if the currents are anti-parallel.

(ii) Attractive if the currents are parallel.

This force per unit length on either conductor is given by $F = \dfrac{\mu_0}{2\pi}\dfrac{I_1 I_2}{r}$.

Where r = perpendicular distance between the parallel conductors

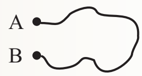
Magnetic Torque On a Closed current Circuit :

When a plane closed current circuit of 'N' turns and of area 'A' per turn carrying a current I is placed in uniform magnetic field, it experience a zero net force, but experience a torque given by $\vec{\tau} = NI\vec{A}\times\vec{B} = \vec{M}\times\vec{B} = BINA\sin\theta$

where $\vec{A}$ = area vector outward from the face of the circuit where the current is anticlockwise, $\vec{B}$ = magnetic induction of the uniform magnetic field.

$\vec{M}$ = magnetic moment of the current circuit = $NI\vec{A}$

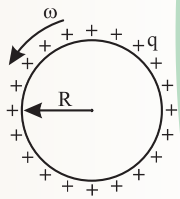
FORCE ON A RANDOM SHAPED CONDUCTOR IN A UNIFORM MAGNETIC FIELD



- Magnetic force on a closed loop in a uniform $\vec{B}$ is zero
- Force experienced by a wire of any shape is equivalent to force on a wire joining points A & B in a uniform magnetic field.

MAGNETIC MOMENT OF A ROTATING CHARGE

If a charge q is rotating at an angular velocity ω, its equivalent current is given as $I = \frac{q\omega}{2\pi}$ & its magnetic moment is $M = I\pi R^2 = \frac{1}{2}q\omega R^2$.



NOTES

The ratio of magnetic moment to angular momentum of a uniform rotating object which is charged uniformly is always a constant. Irrespective of the shape of conductor M/L = q/2m.

- Magnetic dipole
  - Magnetic moment $M = m \times 2l$ where m = pole strength of the magnet
  - Magnetic field at axial point (or End-on) of dipole $\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{M}}{r^3}$
  - Magnetic field at equatorial position (Broad-on) of dipole

    $= \vec{B} = \frac{\mu_0}{4\pi}\frac{\left(-\vec{M}\right)}{r^3}$

- At a point which is at a distance r from dipole midpoint and making angle θ with dipole axis.

$$\text{Magnetic Potential } V = \frac{\mu_0}{4\pi}\frac{M\cos\theta}{r^2}$$

$$\text{Magnetic field } B = \frac{\mu_0}{4\pi}\frac{M\sqrt{1+3\cos^2\theta}}{r^3}$$

- Torque on dipole placed in uniform magnetic field $\vec{\tau} = \vec{M}\times\vec{B}$
- Potential energy of dipole placed in an uniform field $U = -\vec{M}\cdot\vec{B}$
- Intensity of magnetisation $I = M/V$
- Magnetic induction $B = \mu H = \mu_0(H + I)$
- Magnetic permeability $\mu = \frac{B}{H}$
- Magnetic susceptibility $\chi_m = \frac{1}{H} = \mu - 1$
- Curie Law
  For paramagnetic materials $\chi_m \propto \frac{1}{T}$
- Curie Wiess law
  For Ferromagnetic materials $\chi_m \propto \frac{1}{T - T_C}$
  Where T_C = Curie temperature

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CHAPTER 19

Electromagnetic Induction

- When $\vec{B}$ is uniform

  Magnetic flux $\phi = \vec{B}.\vec{A} = BA\cos\theta$
- When $\vec{B}$ is variable

  Magnetic flux $\phi = \int d\phi = \int \vec{B}.\vec{dA} = \int B\cos\theta dA$
- Average induced emf = $\bar{e} = \dfrac{-\Delta\phi}{\Delta t} = -\left[\dfrac{\phi_2 - \phi_1}{t_2 - t_1}\right]$
- Instantaneous induced emf $e_{(t)} = -\dfrac{d}{dt}\phi_{(t)}$ (– is due to Lenz's Law)
- Motional emf = $B_{\perp}v_{\perp}l$
- Motional emf $e = \int de = \int(\vec{v}\times\vec{B}).\vec{dl} = (\vec{v}\times\vec{B}).\vec{l}$
- $\int \vec{E}.\vec{dl} = \dfrac{-d\phi_B}{dt}$ — $\vec{E} \rightarrow$ electric (induced) field,

  $\phi_B \rightarrow$ Magnetic flux
- $\phi_B = LI$ — $L \rightarrow$ self inductance of the coil
- $e = -L\dfrac{dI}{dt}$
- $L = (\mu_0\mu_r N^2 A)/l$ — $L \rightarrow$ self inductance of a coil
- $\phi_2 = MI_1$ and $e_2 = -M\dfrac{dI_1}{dt}$ — M (Mutual inductance)
- The emf induced (in dynamo) $e_{(t)} = BA\omega.\sin\omega t$
- Mutual inductance $M = (\mu_0\mu_r N_1 N_2 A)/l$

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CHAPTER 20

Alternating Current

- The constant value of dc which produces same heat through a resistive element, as due to the alternating current, is known as root mean square value of ac.
- 240 V ac is the rms value of ac voltage. The amplitude of this voltage is $V_M = 240 \times \sqrt{2} = 340$ volt.
- The power rating in ac circuit is the average power rating.
- Power consumed in a circuit is non negative.
- Phase relationships in a.c. circuits is best represented by phasor diagram. A phasor is a vector which rotates with the angular velocity ω. The magnitude of phasor is the peak value of voltage or current (V_o or I_o).
- In purely resistive AC circuit, voltage and current are in the same phase $V = V_o \sin\omega t$ and $I = I_o \sin\omega t$, where $I_o = \frac{V_o}{R}$.
- In purely resistive circuit, average power loss $= I^2_{rms} \times R$, $I_{rms} = \frac{I_o}{\sqrt{2}}$, similarly $V_{rms} = \frac{V_o}{\sqrt{2}}$
- The only element which dissipates energy in ac circuit is resistor (R).
- In purely inductive circuit, inductive reactance $X_L = 2\pi fL = \omega L$ voltage is ahead of current by $\frac{\pi}{2}$, $V = V_o \sin\omega t$, $I = I_o \sin\left(\omega t - \frac{\pi}{2}\right)$, $I_o = \frac{V_o}{X_L}$.
  In this circuit, average power loss = 0.
- In purely inductive or capacitive circuit, $\cos\phi = 0 \Rightarrow \phi = \frac{\pi}{2}$. Average power loss is zero. Although current is flowing in the circuit. Such a current is known as wattless current.

- In AC L-R circuit, total voltage $V = \sqrt{V_R^2 + V_L^2}$.
- In purely capacitive A/C circuit, capacitive reactance $X_C = \frac{1}{2\pi f} = \frac{1}{\omega}$. The current leads the applied voltage by $\frac{\pi}{2}$ or 90º. $V = V_o \sin\omega t$, $I = I_o \sin\left(\omega t + \frac{\pi}{2}\right)$, $I_o = \frac{V_o}{X_c} = 2\pi f V_o$, $V = V_o \sin\omega t$, $I = I_o \sin\left(\omega t + \frac{\pi}{2}\right)$, $I_o = \frac{V_o}{X_c} = 2\pi f V_0$. The average power loss per cycle is ZERO.
- In AC C-R circuit, total voltage $V = \sqrt{V_R^2 + V_C^2}$.
- A circuit containing an inductor L and a capacitor (initially charged) with no ac source and no resistors, exhibits free oscillations. The charge of the capacitor is given by the differential equation $\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$. The sum of energy of capacitor and inductor is constant.
- For a given RLC circuit driven by voltage $V = V_o \sin\omega t$, the current is given by $I = I_o \sin(\omega t + \phi)$ where $I_o = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}$ and $\phi = \tan^{-1}\frac{X_C - X_L}{R}$, impedance $z = \sqrt{R^2 + (X_C - X_L)^2}$.
- The phase difference between voltage across L and voltage across capacitor C, is 180º Thus $V_{LC} = V_L - V_C$.
- The voltage in series LCR A/C circuit is given by $V = \sqrt{V_R^2 + (V_L - V_C)^2}$.
- The average power consumed $= V_{rms} \times I_{rms} \times \cos\phi$, where $\cos\phi$ is the power factor.
- In series LCR circuit, at resonance, $X_L = X_C$, the impedance Z is minimum and equal to R. In this case, the source frequency $\omega = \frac{1}{\sqrt{LC}}$ which equals resonant frequency.
- The quality factor $Q = \omega_0 \frac{L}{R} = \frac{1}{\omega_0 CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$ is an indicator or "sharpness of resonance."
- The power factor in a RLC circuit is a measure of how close the circuit is to consuming maximum power.
- A step up transformer converts low ac voltage to high ac voltage but reduces the current.
- A step down transformer converts high ac voltage to a low ac voltage but increases the currents accordingly.

- In transformer, the primary and secondary voltage are given by $V_S = \left(\dfrac{N_S}{N_P}\right) V_P$ and the current are given by $I_S = \left(\dfrac{N_P}{N_S}\right) I_P$. In step up transformer, $N_S > N_P$ and step down transformer $N_S < N_P$.
- A generator converts mechanical energy into electrical energy, whereas an electric motor converts electrical energy into mechanical energy.
- A transformer does not violate the law of conservation of energy. A step up transformer changes low voltage to a high voltage but reduces the current in the same proportion.

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CHAPTER 21

Electromagnetic Waves

- Four Maxwell's Equations

1. $\oint \vec{E}.\vec{ds} = \dfrac{q}{\varepsilon_0}$

2. $\oint \vec{B}.\vec{ds} = 0$

3. $\oint \vec{E}.\vec{dl} = -\dfrac{d}{dt}\phi_B = \dfrac{-d}{dt}\int \vec{B}.\vec{ds}$

4. $\oint \vec{B}.\vec{dl} = -\mu_0(I_C + I_D) = \mu_0\left(I_C + \varepsilon_0 \dfrac{d\phi_E}{dt}\right)$

- Displacement current $I_D = \varepsilon_0 \dfrac{d}{dt}\phi_E = \varepsilon_0 \dfrac{d\oint \vec{E}.\vec{ds}}{dt} = \dfrac{CdV}{dt}$
- $E_y = E_0 \sin(\omega t - kx)$ and $B_z = B_0 \sin(\omega t - kx)$
- $C_{vacuum} = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$; $c_{medium} = \dfrac{1}{\sqrt{\mu_r \mu_0 \varepsilon_r \varepsilon_0}}$
- $\dfrac{E_0}{B_0} = \dfrac{E_{RMS}}{B_{RMS}} = \dfrac{E}{B} = c$
- Average intensity of wave I_{av} = average energy density (speed of light) of

  $I_{av} = U_{av} \cdot c = \dfrac{E_0 B_0}{2\mu_0} = \dfrac{E_0^2}{2c\mu_0} = \dfrac{cB_0^2}{2\mu_0}$
- Instantaneous energy density $u = \dfrac{1}{2}\varepsilon_0 E^2 + \dfrac{B_0^2}{2\mu_0} = \varepsilon_0 E^2 = \dfrac{B^2}{\mu_0}$
- Average energy density $u_{av} = \dfrac{1}{4}\varepsilon_0 E_0^2 + \dfrac{B_0^2}{4\mu_0} = \dfrac{\varepsilon_0 E_0^2}{2} = \dfrac{B_0^2}{2\mu_0}$

- Energy = (momentum). c or U = Pc
- Radiation pressure $= \dfrac{\text{Intensity}}{c}$ (when the wave is completely absorbed)

  $= \dfrac{2(\text{Intensity})}{c}$ (when the wave is completely reflected)
- Intensity of wave from a source at a distance r from it is proportional to

  $\dfrac{1}{r^2}$ (for a point source)

  $\dfrac{1}{r}$ (for a line source)

  For a plane source intensity is constant & independent of r.

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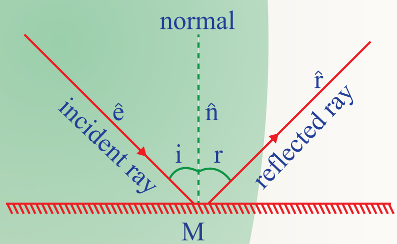
CHAPTER 22

Ray Optics and Optical Instruments

REFLECTION

LAWS OF REFLECTION

- The incident ray the reflected ray and normal to the surface of reflection at the point of incidence lie in the same plane, This plane is called the plane of incidence (also plane of reflection).
- The angle of incidence and the angle of reflection are equal $\angle i = \angle r$



In vector form $\boxed{\hat{r} = \hat{e} - 2(\hat{e}.\hat{n})\hat{n}}$

Object

Real: Point from which rays actually diverge.

Virtual: Point towards which rays appear to converge

Image

Image is decided by reflected or refracted rays only. The point image for a mirror is that point towards which the rays reflected from the mirror, actually converge (real image).

OR

From which the reflected rays appear to diverge (virtual image).

CHARACTERISTICS OF REFLECTION BY A PLANE MIRROR

The size of the image is the same as that of the object.

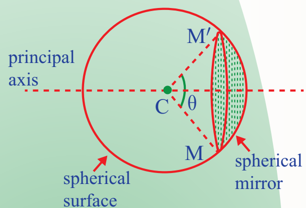
For a real object the image is virtual and for a virtual object the image is real.

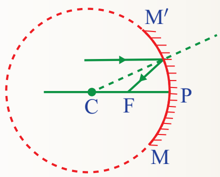
For a fixed incident light ray, if the mirror be rotated through an angle θ the reflected ray turns through an angle 2θ in the same sense.

Number of images (n) in inclined mirror Find $\frac{360}{\theta} = m$

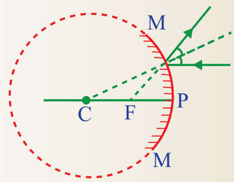
- If m is even, then n = m – 1, for all positions of object.
- If m is odd, then n = m, If object is not on bisector and n = m – 1, If object at bisector
- If m is fraction then n = nearest even number

Spherical Mirrors





concave mirror



convex mirror

Mirror Formula : $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$.

f = x-coordinate of focus

v = x-coordinate of image

u = x-coordinate of object

Note : Valid only for paraxial rays.

Transverse Magnification : $m = \frac{h_2}{h_1} = -\frac{v}{u}$

h_2 = y co-ordinate of image

h_1 = y co-ordinate of the object

(both perpendicular to the principal axis of mirror)

Longitudinal magnification : m_2

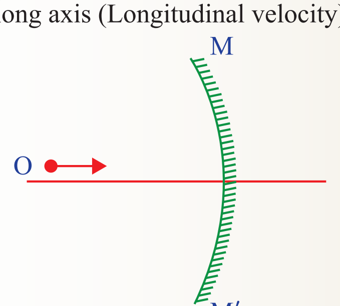
$$m_2 = \frac{\text{Length of image}}{\text{Length of object}}$$

for small object $m_2 = -m_t^2$

m_t = transverse magnification.

Velocity of image of Moving Object (Spherical Mirror)

Velocity component along axis (Longitudinal velocity)



When an object is coming from infinite towards the focus of concave mirror

$$\because \frac{1}{v}+\frac{1}{u}=\frac{1}{f} \qquad \therefore -\frac{1}{v^2}\frac{dv}{dt}-\frac{1}{u^2}\frac{du}{dt}=0 \Rightarrow \vec{v}_{IM}=-\frac{v^2}{u^2}\vec{v}_{OM}=-m^2\vec{v}_{OM}$$

- $V_{IM}=\frac{dv}{dt}=$ velocity of image with respect to mirror
- $V_{OM}=\frac{du}{dt}=$ velocity of object with respect to mirror.

Newton's Formula :

Applicable to a pair of real object and real image position only. They are called conjugate positions or foci, X_1, X_2 are the distance along the principal axis of the real object and real image respectively from the principal focus

$$X_1X_2 = f^2$$

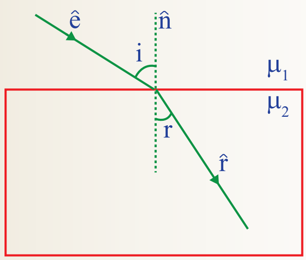
Optical Power : Optical power of a mirror (in Diopters) $=-\frac{1}{f}$

where f = focal length (in meters) with sign.

REFRACTION-PLANE SURFACE

Laws of Refraction (At any Refracting Surface)

(i) Incident ray, refracted ray and normal always lie in the same plane.



In vector form $(\hat{e}\times\hat{n}).\hat{r}=0$

(ii) The product of refractive index and sine of angle of incidence at a point in a medium is constant. $\mu_1 \sin i = \mu_2 \sin r$ (Snell's law)

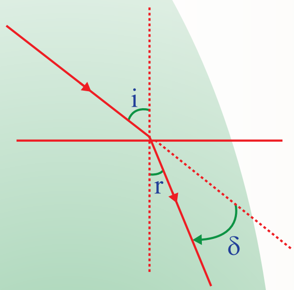
Snell's Law

$$\boxed{\frac{\text{Sin } i}{\text{Sin } r}={}_1n_2=\frac{n_2}{n_1}=\frac{v_1}{v_2}=\frac{\lambda_1}{\lambda_2}}$$ In vector form $\boxed{\mu_1\,|\hat{e}\times\hat{n}|=\mu_2\,|\hat{r}\times\hat{n}|}$

NOTES

Frequency of light does not change during refraction.

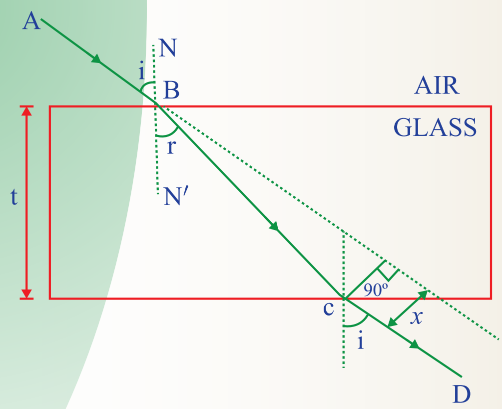
Deviation of a Ray due to refraction



$\boxed{\text{angle of deviation}, \delta = i - r}$ (clockwise)

Refraction Through a Parallel Slab

Emerged ray is parallel to the incident ray, if medium is same on both sides.

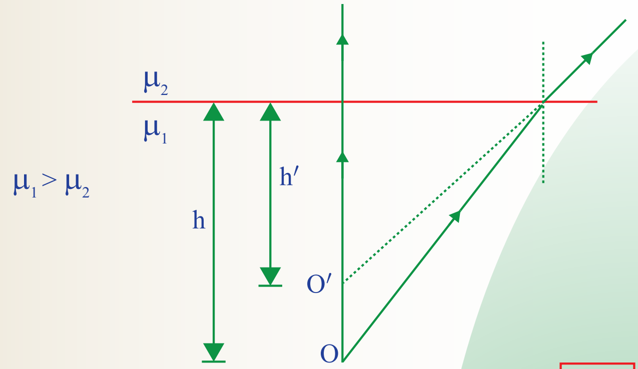


Lateral shift $x = \dfrac{t\sin(i - r)}{\cos r}$; t = thickness of slab

NOTES

Emergent ray will not be parallel to the incident ray if the medium on both the sides are different.

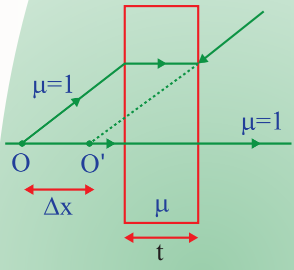
Apparent Depth of Submerged Object : (h' < h)



For near normal incidence $h' = \frac{\mu_2}{\mu_1} h$

Δx = Apparent shift $= t\left(1 - \frac{1}{\mu}\right)$

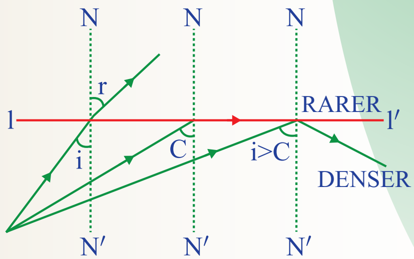
always in direction of incident ray.



NOTES

h and h' are always measured from surface.

Critical Angle & Total Internal Reflection (TIR)

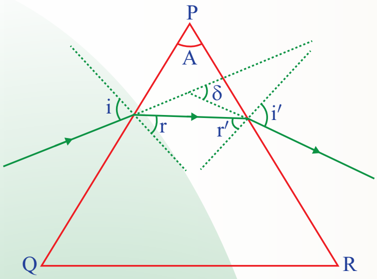


Conditions of TIR

- Ray is going from denser to rarer medium
- Angle of incidence should be greater than the critical angle (i > C).

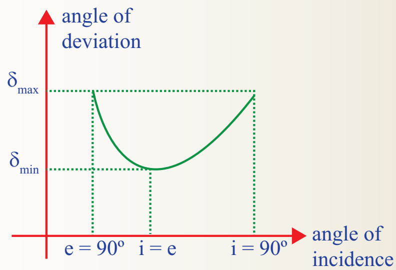
Critical angle $C = \sin^{-1}\frac{\mu_R}{\mu_D} = \sin^{-1}\frac{v_D}{v_R} = \sin^{-1}\frac{\lambda_D}{\lambda_R}$

Refraction Through Prism :



- $\delta = (i + i') - (r + r')$
- $r + r' = A$
- $\delta = i + i' - A$

- Variation of δ versus i



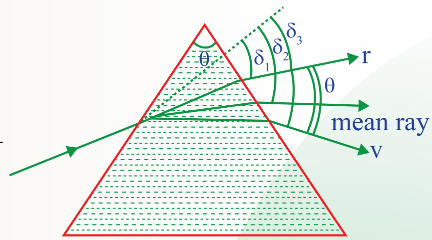
- There is one and only one angle of incidence for which the angle of deviation is minimum. When $\delta = \delta_{min}$ then $i = i'$ & $r = r'$, the ray passes symetrically about the prism, & then $n = \dfrac{\sin\left[\dfrac{A+\delta m}{2}\right]}{\sin\left[\dfrac{A}{2}\right]}$, where n = absolute R.I. of glass.

NOTES

When the prism is dipped in a medium then n = R.I. of glass w.r.t. medium.

- For a thin prism ($A \le 10^{o}$) ; $\delta = (n-1)A$
- **Dispersion of Light :** The angular splitting of a ray of white light into a number of components when it is refracted in a medium other than air is called **Dispersion of Light.**
- **Angle of Dispersion :** Angle between the rays of the extreme colours in the refracted (dispersed) light is called Angle of Dispersion. $\theta = \delta_v - \delta_r$
- Dispersive power (ω) of the medium of the material of prism.

$$\omega = \frac{\text{angular dispersion}}{\text{devitation of mean ray (yellow)}}$$



For small angled prism ($A \le 10°$); $\omega = \frac{\delta_v - \delta_R}{\delta_y} = \frac{n_v - n_R}{n - 1}; n = \frac{n_v + n_R}{2}$

n_v, n_R & n are R.I. of material for violet, red & yellow colours respectively.

Refraction At Spherical Surface

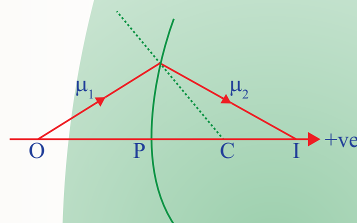
(a) $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

v, u & R are to be kept with sign as

v = PI

u = –PO

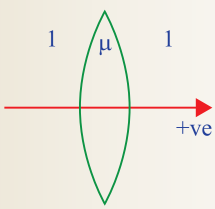
R = PC



(b) $m = \frac{\mu_1 v}{\mu_2 u}$

Lens Formula

(a)



$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ **(b)** $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ **(c)** $m = \frac{v}{u}$

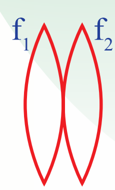
Power of Lenses

Reciprocal of focal length in meter is known as power of lens.

SI unit : dioptre (D)

Power of lens : $P = \frac{1}{f\,(m)} = \frac{100}{f\,(cm)}$ dioptre

Combination of Lenses

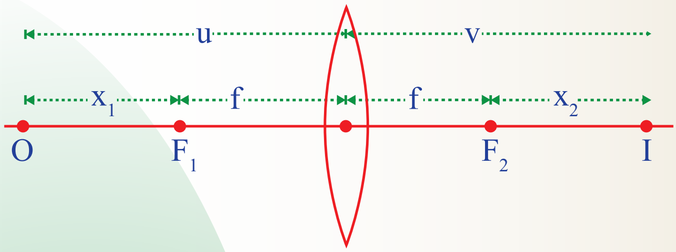


Two thin lens are placed in contact to each other

Power of combination. $P = P_1 + P_2 \Rightarrow \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

Use sign convention while solving numericals.

Newton's Formula

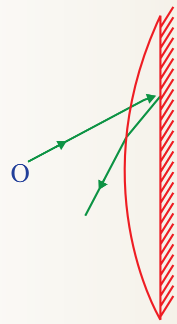


$$\boxed{f = \sqrt{x_1 x_2}}$$

x_1 = distance of object from focus; x_2 = distance of image from focus.

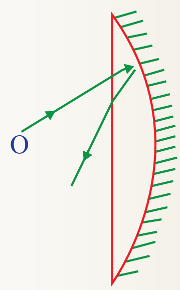
Silvering of one surface of lens (use $P_{eq} = 2P_l + P_m$)

- When plane surface is silvered



$$f = \frac{R}{2(\mu - 1)}$$

- When convex surface is slivered



$$f = \frac{R}{2\mu}$$

OPTICAL INSTRUMENTS

For Simple microscope

- Magnifying power when image is formed at D : $M = 1 + D/f$
- When image is formed at infinity $M = D/f$

For Compound microscope

- When image is formed at infinity $M = -\frac{v_0}{u_0}\left(\frac{D}{u_e}\right)$

- Magnifying power when final image is formed at D, $M = -\frac{v_0}{u_0}\left(1+\frac{D}{f_e}\right)$
- Tube length $L = v_0 + |u_e|$
- When final images is formed at infinity $M = -\frac{v_0}{u_0} \times \frac{D}{f_e}$ and $L = v_0 + f_e$

Astronomical Telescope : $M = -\frac{f_0}{u_e}$

- Magnifying power when final image is formed at D: $M = \frac{f_0}{f_e}\left(1+\frac{D}{f_e}\right)$
- Tube length : $L = f_0 + |u_e|$
- When final image is formed at infinity : $M = \frac{f_0}{f_e}$ and $L = f_0 + f_e$

Limit of resolution for microscope : $\frac{1.22\lambda}{2a\sin\theta} = \frac{1}{\text{Resolving power}}$

Limit of resolution for telescope : $\frac{1.22\lambda}{a} = \frac{1}{\text{Resolving power}}$

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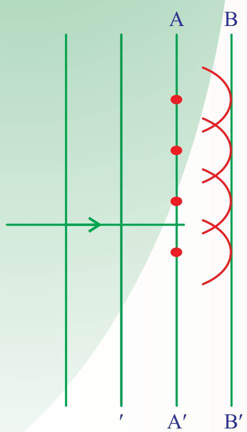
CHAPTER 23

Wave optics

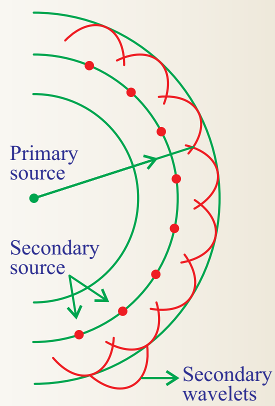
HUYGEN'S WAVE THEORY

- Each point source of light is a center of disturbance from which waves are emitted in all directions. The locus of all the particles of the medium oscillating in the same phase at a given instant is called a wavefront.
- Each point on a wave front is a source of new disturbance, called secondary wavelets. These wavelets are spherical and travel with speed of light in that medium.
- The forward envelope of the secondary wavelets at any instant gives the position of the new wavefront.
- In homogeneous medium, the wave front is always perpendicular to the direction of wave propagation.

Plane wavefront



Spherical wavefront



COHERENT SOURCES

Two sources are coherent if and only if they produce waves of same frequency (and hence wavelength) and have a constant initial phase difference.

INCOHERENT SOURCES:

Two sources are said to be incoherent if they have different frequency and initial phase difference varies with time.

INTERFERENCE : YDSE

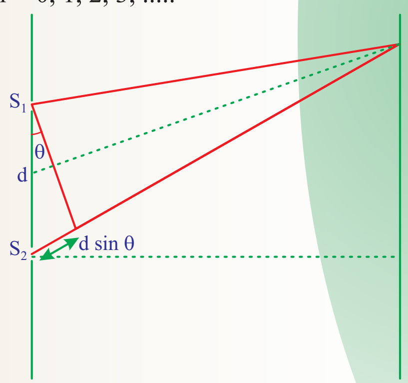
- Resultant intensity for coherent sources $I = I_1 + I_2 + \sqrt{I_1 I_2} \cos\phi_0$
- Resultant intensity for incoherent sources $I = I_1 + I_2$
- Intensity $\propto$ width of slit $\propto$ (amplitude)^2

$$\Rightarrow \frac{I_1}{I_2} = \frac{W_1}{W_2} = \frac{A_1^2}{A_2^2} \Rightarrow \frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2$$

- Distance of nth bright fringe $y_n = \dfrac{n\lambda D}{d}$

  Path difference = $n\lambda$

  where n = 0, 1, 2, 3, ...



- Distance of mth dark fringe

$$y_m = \frac{(2m+1)\lambda D}{2d}$$

  Path difference = $(2m + 1)\dfrac{\lambda}{2}$ where m = 0, 1, 2, 3,.....

- Fringe width $\beta = \dfrac{\lambda D}{d}$
- Angular fringe width = $\dfrac{\beta}{D} = \dfrac{\lambda}{d}$

- If a transparent sheet of refractive index μ and thickness t is introduced in one of the paths of interfering waves, optical path will become 'μt' instead of 't'. Entire fringe pattern shifts by $\frac{D[(\mu-1)t]}{d} = \frac{\beta}{\lambda}(\mu-1)t$ towards the side in which the thin sheet is introduced without any change in fringe width.

$$I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

DIFFRACTION

- In Fraunhofer diffraction
  - For minima $\quad a \sin\theta_n = n\lambda$
  - For maxima $\quad a \sin\theta_n = (2n+1)\frac{\lambda}{2}$
  - Linear width of central maxima $\quad W = \frac{2\lambda D}{a}$
  - Angular width of central maxima $\quad W_\theta = \frac{2\lambda}{a}$

POLARIZATION

Brewster's law

$$\mu = \tan\theta_p \Rightarrow \theta_p = \tan^{-1}\mu$$

$\theta_p \rightarrow$ polarization or Brewster's angle

Here reflecting and refracting rays are perpendicular to each other.

Malus law

$$I = I_0 \text{Cos}^2\,\theta$$

$I_0 \rightarrow$ Maximum intensity of polarized light.

CHAPTER 24

Dual Nature of Radiation & Matter

Photon: These are packets of energy and travel in a straight line. Velocity in different media is different but frequency of photon in different media is same. Its momentum is $\frac{h}{\lambda}$.

Dual nature of light and matter: Light exhibits particle aspects in certain phenomenon (e.g., photoelectric effect, emission and absorption of radiation), while wave aspects in other phenomena (e.g., interference, diffraction and polarization i.e., light possesses dual nature.

De Broglie concluded that matter also possess dual nature, Light and matter both possess properties of matter and wave.

Work function: Minimum energy required by a free electron to just come out of the metal surface (with KE = 0) is called work function of the metal. Work function is expressed in eV.

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

Photoelectric effect: The phenomenon of emission of electrons from a metallic surface by the use of light (or radiant) energy is called photoelectric effect, as it has low work function.

Threshold frequency : The minimum frequency of incident light which is just capable of ejecting electrons from a metal is called the threshold frequency. It is denoted by ν_0. The corresponding wavelength of light is called threshold wavelength (λ_0). If $\nu < \nu_o$, there will be no photo-electric emission.

Stopping potential: The minimum retarding potential applied to anode of a photoelectric tube which is just capable of stopping photoelectric current. It is denoted by V_0 (or V_s). Stopping potential depends upon the frequency of incident light i.e., $V_0 \propto \nu$.

De Broglie hypothesis: A wave is associated with a moving material particle which controls the particle in all respect. The wavelength associated with a moving particle is given by $\lambda = \frac{h}{mv}$ where m is the mass of the particle moving with v velocity and h is plank's constant. This wave is called de Broglie wave.

KEY TIPS

- Einstein's photoelectric cell equation, $\frac{1}{2}mv_{max}^2 = h\nu - h\nu_0$
- Work function and threshold frequency or threshold wavelength, $\phi_0 = h\nu_0 = \frac{hc}{\lambda_0}$
- Energy of photon, $E = h\nu = \frac{hc}{\lambda}$
- Momentum of photon, $p = \frac{E}{c} = \frac{h}{\lambda}$
- De Broglie wavelength of a material particle, $\lambda = \frac{h}{mv}$
- De Broglie wavelength of an electron accelerated through a potential V volt, $\lambda = \frac{12.27}{\sqrt{V}}\text{Å} = \frac{1.227}{\sqrt{V}}\text{nm}$
- De Broglie wavelength of a particle in terms of temperature (T), $\lambda = \frac{h}{\sqrt{3mkT}}$
- De Broglie wavelength in terms of energy of a particle (E), $\lambda = \frac{h}{\sqrt{2mE}}$

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CHAPTER 25

Atoms

SOME IMPORTANT DEFINITIONS

Impact parameter: Perpendicular distance of initial velocity vector of α-particles from the centre of the nucleus.

Distance of closest approach: Distance of a point from nucleus at which α-particle is nearest to the centre of nucleus.

Bohr radius: First orbit of hydrogen atom, called Bohr radius. (a_0)

Ground state : Lowest state of atom, called the ground state, is the state in which electron revolves in the orbit of smallest radius, the Bohr radius, a_0.

Ionization energy: Minimum energy required to free an electron from the ground state of hydrogen atom is called the ionization energy.

BOHR'S MODEL OF HYDROGEN ATOM

POSTULATES

(a) $$\frac{\theta^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$$

(b) $$mvr = \frac{nh}{2\pi}$$

(c) $$E_i - E_f = hv = \frac{hc}{\lambda}$$

Radius of nth orbit, $$r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m e^2}$$

Orbital speed, $$v_n = \frac{nh}{2\pi m r_n}$$

Energy of nth orbit, $E_n = -\left(\frac{me^4}{8\varepsilon_0^2 h^2}\right)\frac{1}{n^2} = -\frac{13.6}{n^2}eV$

Total Energy = –Kinetic Energy

Potential Energy = 2 × Total Energy

Rydberg's constant

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$R = \frac{me^4}{8\varepsilon_0^2 h^3 c}$$

$n_1 = 1, n_2 = 2, 3, ...$ for Lyman series

$n_1 = 2, n_2 = 3, 4, ...$ for Balmer series

$n_1 = 3, n_2 = 4, 5, ...$ for Paschen series

$n_1 = 4, n_2 = 5, 6, ...$ for Brackett series

$n_1 = 5, n_2 = 6, 7, ...$ for Pfund series

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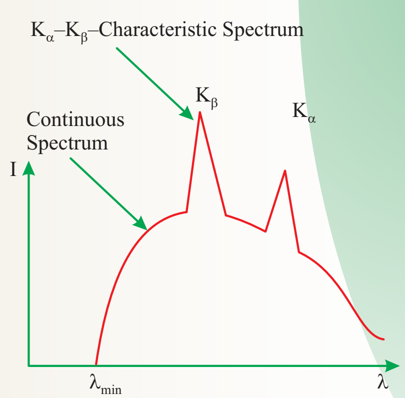
CHAPTER 26

Nuclei

X-RAYS

- X-rays are produced by bombarding high speed electrons on a target of high atomic weight and high melting point.
- Are short wavelength (0.1 Å to 10 Å) of electromagnetic radiation.
- Are produced when a metal anode is bombarded by highly energetic electrons
- Are not affected by electric and magnetic field.
- They cause photoelectric emission.



Characteristics equation $\qquad eV = h\nu_{max} = \dfrac{hc}{\lambda_{min}}$

e = electronic charge;

V = accelerating potential

ν_{max} = maximum frequency of X-Rays

- Intensity of X - rays depends on number of electrons hitting the target.
- Cut off wavelength or minimum wavelength, where V (in volts) is the potential difference applied to the tube $\lambda_{min} \cong \frac{12400}{V}$ Å
- Continuous spectrum due to retardation of electrons.

CHARACTERISTIC X-RAYS

For K_α, $\lambda = \frac{hc}{E_K - E_L}$ For K_β, $\lambda = \frac{hc}{E_L - E_M}$

NUCLEAR COLLISION

We can represent a nuclear collision or reaction by the following notation, which means X (a,b) Y

$$\underset{\text{(bombarding particle)}}{a} + \underset{\text{(at rest)}}{X} \rightarrow Y + b$$

The following can be applied:

(i) Conservation of momentum (ii) Conservation of charge

(iii) Conservation of mass-energy

For any nuclear reaction $\underset{K_1}{a} + \underset{K_2}{X} \rightarrow \underset{K_3}{Y} + \underset{K_4}{b}$

By mass-energy conservation

(i) $K_1 + K_2 + (m_a + m_x)c^2 = K_3 + K_4 + (m_y + m_b)c^2$

(ii) Energy released in any nuclear reaction or collision is called Q value of the reaction.

(iii) $Q = (K_3 + K_4) - (K_1 + K_2) = \Sigma Kp - \Sigma K_R = (\Sigma m_R - \Sigma m_p)c^2$

where $p \rightarrow$ products and $r \rightarrow$ reactants

(iv) If Q is positive, energy is released and products are more stable in comparison to reactants.

(v) If Q is negative, energy is absorbed and products are less stable in comparison to reactants.

$$Q = \Sigma(\text{B.E.})_{\text{product}} - \Sigma(\text{B.E.})\ \text{reactants}$$

NUCLEAR FISSION

By bombarding a particle on a heavy nucleus (A > 230), it splits into two or more light nuclei. In this process certain mass disappears which is obtained in the form of energy (enormous amount)

$A + p \rightarrow B + C + Q$

Nuclear Fusion :

It is the phenomenon of fusing two or more light nuclei to form a single heavy nucleus.

$$A + B \rightarrow C + Q\ (\text{Energy})$$

The product (C) is more stable then reactants (A and B) and $m_c < (m_a + m_b)$ and mass defect $\Delta m = [(m_a + m_b) - m_c]$ amu

Energy released is $E = (\Delta m)\ 931$ MeV

RADIOACTIVITY

- **Radioactive Decays :** Generally, there are three types of radioactive decays

  (i) α decay (ii) β^- and β^+ decay (iii) γ decay
- α **decay :** By emitting α particle, the nucleus decreases it's mass number and move towards stability. Nucleus having $A > 210$ shows α decay.
- β **decay :** In beta decay, either a proton is converted into neutron or neutron is converted into proton.
- γ **decay :** When an α or β decay takes place, the daughter nucleus is usually in higher energy state, such a nucleus comes to ground state by emitting a photon or photons.

  Order of energy of γ photon is 100 keV

LAWS OF RADIOACTIVE DECAY

- The rate of disintegration is directly proportional to the number of radioactive atoms present at that time i.e., rate of decay $\propto$ number of nuclei.

  Rate of decay = λ (number of active nuclei) i.e., $\frac{dN}{dt} = -\lambda N$.

  where λ is called the decay constant.

  This equation may be expressed in the form $\frac{dN}{N} = -\lambda dt$.

  $$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt \Rightarrow \ln\left(\frac{N}{N_0}\right) = -\lambda t$$

  where N_0 is the number of parent nuclei at t = 0. The number that survives at time t is therefore $N = N_0 e^{-\lambda t}$ and $t = \frac{2.303}{\lambda}\log_{10}\left(\frac{N_0}{N_1}\right)$

  $N = N_0 e^{-\lambda t}$ where λ = decay constant

- ❑ Half life $t_{1/2} = \frac{\ln 2}{\lambda}$

- Average life $t_{av} = \frac{1}{\lambda}$
- Activity $R = \lambda N = R_0 e^{-\lambda t}$
- $1Bq = 1$ decay/s,
- 1 curie $= 3.7 \times 10^{10}$ Bq,
- 1 rutherford $= 10^6$Bq
- After n half lives Number of nuclei left $= \frac{N_0}{2^n}$
- Probability of a nucleus for survival of time t $= \frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$

• Equivalence of mass and energy $E = mc^2$

Note :- $1u = 1.66 \times 10^{-27}$ kg $\equiv 931.5$ MeV

• Binding energy of ${}_zX^A$

$BE = \Delta mc^2 = [Zm_p + (A-Z)m_n - m_x]c^2 = [Zm_H + (A-Z)m_n - m_x]c^2$

• Q-value of a nuclear reaction

For $a + X \rightarrow Y + b$ or X (a, b)Y; $Q = (M_a + M_x - M_y - M_b)c^2$

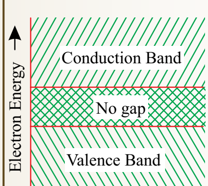
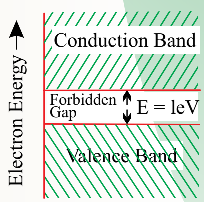
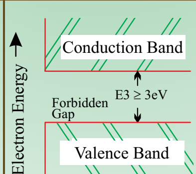
• Radius of the nucleus

$R = R_0 A^{1/3}$ where $R_0 = 1.3\ f_m = 1.3 \times 10^{-15}$ m

CHAPTER 27

Semiconductor and Digital Electronics

Comparison between conductor, semiconductor and insulator

Characteristic	Conductor	Semiconductor	Insulator
Resistivity	$10^{-2} - 10^{-8}$ Ωm	$10^{-5} - 10^{-6}$ Ωm	$10^{-11} - 10^{-19}$ Ωm
Conductivity	$10^{-2} - 10^{-8}$ mho/m	$10^{-5} - 10^{-6}$ mho/m	$10^{-11} - 10^{-19}$ mho/m
Temp. Coefficient of resistance (α)	Positive	Negative	Negative
Current	Due to free electrons	Due electrons and holes	No current
Energy band diagram	 Conductor	 Semi conductor	 Insulator
Forbidden energy gap	0eV	1eV	≥ 3eV
Example	Al, Pt, Cu	Si, Ge, GaAs	diamond, mica,wood, plastic

- Number of electrons reaching from valence band to conduction band

$$n = AT^{3/2}e^{-\frac{\Delta Eg}{2kT}}$$

SEMICONDUCTOR

↓

Intrinsic semiconductor

↓

pure form of Ge, Si

$n_e = n_h = n_i$

Extrinsic semiconductor (doped)

↓

N-type
pentavalent impurity
(As, P, etc.)
donor impurity (N_p)
$n_e >> n_h$

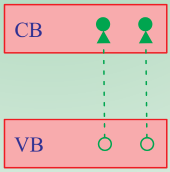
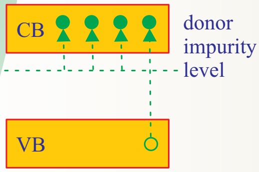
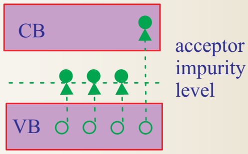
P-type
trivalent impurity
(B, Ga, Al)
acceptor impurity (Na)
$n_h >> n_e$

MASS-ACTION LAW $n_i^2 = n_e \times n_h$

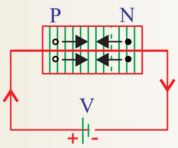
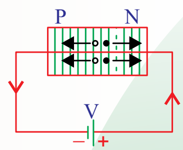
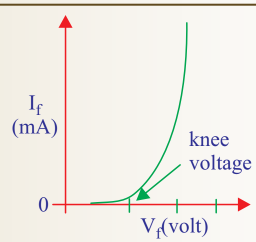
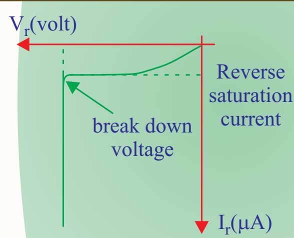
For n-type semiconductor $n_e = N_D$

For P-type semiconductor $n_h = N_A$

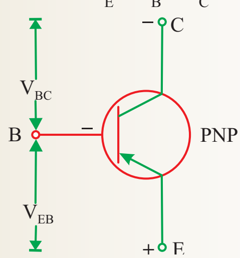
Conductivity $n_i e(\mu_e + \mu_h)$

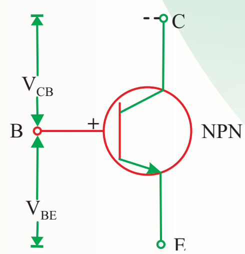
Intrinsic Semiconductor	N-type (Pentavalent impurity)	P-type (Trivalent impurity)
		
Current due to electron and hole	Mainly due to electrons	Mainly due to holes
$n_e = n_h = n_i$	$n_h << n_e$ ($N_D \simeq n_e$)	$n_e >> n_e$ ($N_A \simeq n_h$)
Entirely neutral	Entirely neutral	Entirely neutral
Quantity of electrons and holes are equal	Majority → Electrons Minority → Holes	Majority → Holes Minority → Electrons
$I = I_e + I_h$	$I \simeq I_e$	$I \simeq I_h$

Comparison between Forward Bias and Reverse Bias

Forward Bias	Reverse Bias
	
Potential Barrier reduces	Potential Barrier increases
Width of depletion layer reduces	Width of depletion layer increases
P-N jn. provide very low resistance	P-N jn. provide very high resistance
Forward current flows in the circuit (mA)	Very small current flow. (μA)
Current flows mainly due to majority carriers.	Current flows mainly due to minority carriers.
Forward characteristic curves.	Reverse characteristic curve
	
Forward resistance : $R_f = \frac{\Delta V_f}{\Delta I_f} \cong 10^2\,\Omega$	Reverse resistance : $R_r = \frac{\Delta V_r}{\Delta I_r} \cong 10^6\,\Omega$
Order of knee or cut in voltage	Breakdown voltage
Ge $\rightarrow$ 0.3 V	Ge $\rightarrow$ 25 V
Si $\rightarrow$ 0.7 V	Si $\rightarrow$ 35 V

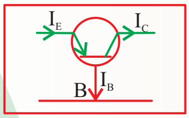
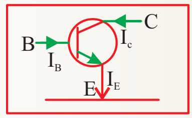
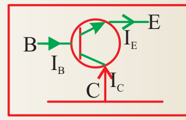
- For transistor $I_E = I_B + I_C$





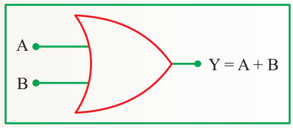
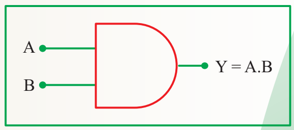
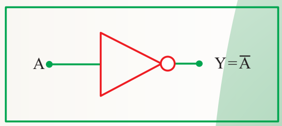
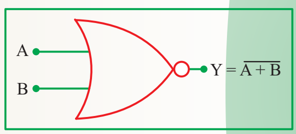
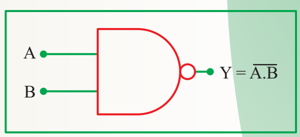
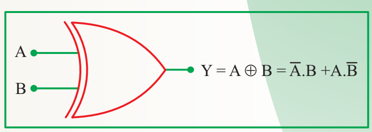
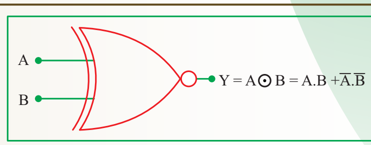
Three Types of transistor configurations:

1. Common Base (CB)
2. Common Emitter (CE)
3. Common Collector (CC)

Characteristic	CB	CE	CC
			
Input Resistance	Low (100 Ω)	High (750 Ω)	Very High ≅ 750 kΩ
Output resistance	Very High	High	Low
Current Gain	α	β	γ
	$\alpha = \frac{I_C}{I_E} < 1$	$\beta = \frac{I_c}{I_B} > 1$	$\gamma = \frac{I_E}{I_B} > 1$
Voltage Gain	$A_v = \frac{V_o}{V_i} = \frac{I_C R_L}{I_E R_i}$	$A_v = \frac{V_o}{V_i} = \frac{I_C R_L}{I_B R_i}$	$A_v = \frac{V_o}{V_i} = \frac{I_E R_L}{I_B R_i}$
	$A_v = \alpha \frac{R_L}{R_i} \cong 150$	$A_v = \beta \frac{R_L}{R_i} \cong 500$	$A_v = \gamma \frac{R_L}{R_i} < 1$
Power Gain	$A_p = \frac{P_{out}}{P_{in}}$ $= \alpha^2 \frac{R_L}{R_i}$	$A_p = \frac{P_{out}}{P_{in}}$ $= \beta^2 \frac{R_L}{R_I}$	$A_p = \frac{P_o}{P_i}$ $= \gamma^2 \frac{R_L}{R_i}$
Phase difference (between output and input)	0	π or 180°	0
Application	For High Frequency	For Audible frequency	For Impedance Matching

Relation between α, β and γ : $\beta = \dfrac{\alpha}{1-\alpha}$, $\gamma = 1 + \beta$ and $\gamma = \dfrac{I}{I-\alpha}$

Logic Gates

OR gate	
AND gate	
NOT gate	n 
NOR gate	
NAND gate	
XOR gate	
XNOR gate	

- De Morgan's theorem $\qquad \overline{A+B} = \overline{A}\cdot\overline{B}$, $\overline{A\cdot B} = \overline{A} + \overline{B}$

CHAPTER 28

Communication System

NOTES

- Communication is the act of transmission and reception of information.
- Two basic modes of communication are point to point and broadcast.
- Noise refers to the unwanted signals that tends to disturb the transmission and processing of message signal in a communication system.
- Attenuation is the loss of strength of a signal while propagating through a medium.
- Amplification is the process of increasing the amplitude (and consequently the strength) of a signal using an electronic circuit called amplifier.
- Range is the largest distance between a source and destination upto which the signal is received with sufficient strength.
- Band-width refers to the frequency range over which an equipment operates or the portion of the spectrum occupied by signal.

KEY TIPS

- The maximum line of sight distance d_M between the two antennas having height h_T and h_R, above the earth, is given by

  $$d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

  Modulation At the transmitter, information contained in the low frequency message, is superimposed on a high frequency wave. This process is known as modulation.
  Three types of modulation are amplitude modulation (AM), frequency modulation (FM) and pulse modulation (PM).

- Modulation index $\mu = \frac{A_m}{A_c}$ where A_m and A_c are the amplitudes of modulating signal and carrier wave.

  De-modulation is the reverse process of modulation. This is the process of extraction of information from the carrier wave.

- In practice $\mu \le 1$ to avoid distortion.

- The amplitude modulated signal consists of three frequencies ω_c, $\omega_c + \omega_m$ and $\omega_c - \omega_m$. Here $\omega_c + \omega_m$ and $\omega_c - \omega_m$ are known as upper side band and lower side band respectively.

- Height of the half wave antenna is equal to $\frac{\pi}{2}$.

- Height of the quarter wave antenna is equal to $\frac{\pi}{4}$.

- In amplitude modulation $P_t = P_c\left[1 + \frac{\mu^2}{2}\right]$

- Maximum modulated frequency that can be detected by diode detector $f_m = \frac{1}{2\pi R\mu}$

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